# Hybrid Serial Concatenated Network Codes for Burst Erasure Channels

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**Abstract**

Information paths in current communication networks can often be modelled by set of serial links interconnected by intermediate nodes. These kind of scenarios are called line networks. If the links between nodes experience connection problems, burst erasures of information can happen. In this article, hybrid serial concatenated network codes are proposed, which consist in the serial concatenation of a 'classical' coding scheme and a network code. A novel way to decrease complexity of this approach is that the 'classical' coding scheme is not performed on a link level. The new coding schemes improve performance of the communications in line networks in terms of error-correcting capability. The evaluations involve MATLAB simulations and analytical analysis.

**Index Terms**

Network error correction (NEC) codes, random linear network coding (RLNC), serial concatenation, burst erasure channels.

## I. Introduction

Network error correction (NEC) codes [1], [2] are network codes, which deal with errors and erasures. They represent an extension of classical time domain codes, in space domain. In 2008, the application of linear NEC (LNEC) codes in packet networks was studied by [3] for global kernel errors. Next, the error-correcting capabilities and the bounds of random LNEC (RLNEC) codes were analysed in details by [4], [5]. An exhaustive study of NEC theory was presented in [6]. Moreover, this article defined a kind of codes, called product codes, obtained by concatenating a NEC code in space domain and a classical error-correcting code in time domain. In particular, that scheme consists in a NEC code to protect the multicast communication and in a systematic error-correcting code (local code) to protect the communication of each link. Side by side, network concatenated codes were also studied by both [7] in 2008, and [8] in 2010. The first investigated the concatenation of a convolutional code (outer code) and a RLNEC code (inner code) for binary erasure channels. The second analysed the concatenation of random linear network coding (RLNC) and Reed-Solomon (RS) codes for additive-white Gaussian noise (AWGN) channels.

Our work focus on the design of optimal hybrid serial concatenated network codes (HSCNCs) in line networks with burst erasure channels (BEC). The term 'hybrid' means that the serial concatenation involves a network code and either a classical erasure code or a fountain code. The main difference with previous results is that our concatenated codes are applied end-to-end. Intermediate nodes can only perform RLNC operations. Therefore, no erasure code is applied at link level. This strategy can guarantee erasure protection by keeping complexity low.

In this paper, two schemes are considered: RS codes and RLNC, and Luby Transform (LT) codes and RLNC, the so called Batched Sparse (BATS) codes [9]. Especially, RS and LT codes have been chosen because they currently represent the most used erasure coding schemes. The main achievement of HSCNCs is the higher error-correcting capability against burst erasures when RS codes are used. At the best of authors' knowledge, HSCNCs in line networks with burst erasure channels have not been studied yet.

The rest of the paper is organised as follows. Section II describes the system model and the notation for RLNEC codes. Next, Section III defines HSCNCs. Finally, Section IV depicts and discusses the simulation results obtained with MATLAB implementation.

## II. System Model and Problem Statement

Figure 1(a) represents the basic case of a line network, in which a source is communicating with a sink via a BEC. In this point-to-point scenario, the source generates and collects a matrix $\mathbf{U}$ of $k$ packets, each of which is constituted by $p$ symbols over a finite field $\mathbb{F}_q$ ($q = 2^m$). If the nodes performs RLNC operations, $\mathbf{U}$ is given as input to a RLNC encoder, which outputs a matrix $\mathbf{X}$ of $n$ encoded rows ($n \geq k$). The structure of $\mathbf{X}$ is

$$\mathbf{X} = \begin{bmatrix} \mathbf{C} & \tilde{\mathbf{X}} \end{bmatrix} \tag{1}$$

where $\mathbf{C}$ is the matrix of coefficients of the linear combinations of the rows of $\mathbf{U}$ and $\tilde{\mathbf{X}}$ is the matrix of the codewords. Since RLNC is used as an error-correcting code, we define $r = k/n$ as the rate of the code. Side by side, the *redundancy* introduced by the code, is $n - k$ packets.

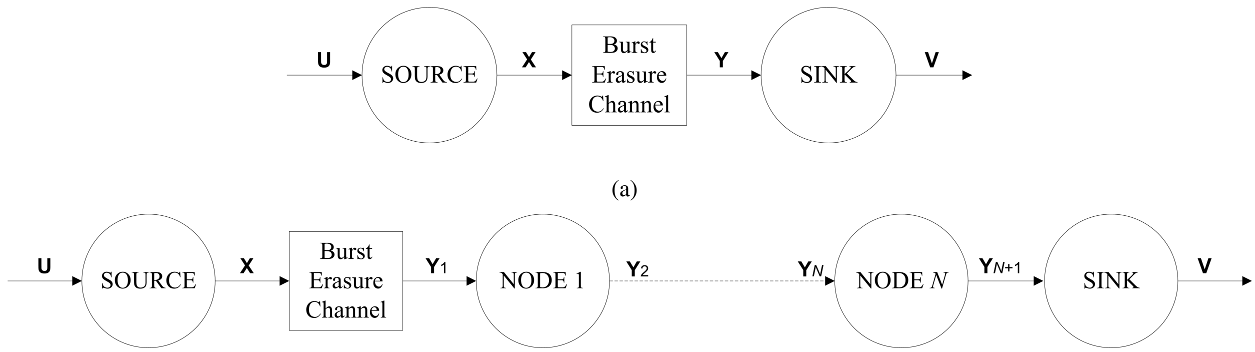


Fig. 1. (a) Point-to-point scenario (line network with no intermediate nodes), in which the encoder encodes source packets of matrix $\mathbf{U}$ into the encoded packets of matrix $\mathbf{X}$. Next, the encoded packets are sent to the destination via a burst erasure channel. Finally, decoder receives matrix $\mathbf{Y}$ and tries to recover the erased information (b) Generalised line network, in which the encoder encodes source packets of matrix $\mathbf{U}$ into the encoded packets of matrix $\mathbf{X}$. The encoded packets are sent to the first intermediate node (NODE 1) via a burst erasure channel. Next, each one of the $N$ intermediate nodes is receiving a matrix $Y_{j-1}$ and re-encoding it into a matrix $\mathbf{Y}_{j+1}$. Finally, decoder receives matrix $\mathbf{Y}_{N+1}$ and tries to recover the erased information.

After the encoding process, the source sends the matrix $\mathbf{X}$ to the BEC: the capacity of the channel is enough to send $n$ packets per unit of time. The BEC sequentially and randomly deletes $\rho$ rows and its burst erasure probability is defined as $b_e$: in particular, this probability is the ratio $l_B/np$, where $l_B$ is the length of the burst. The output of this channel is described by a matrix $\mathbf{Y}$ with $n - \rho$ rows.

Next, $\mathbf{Y}$ is passed to the RLNC decoder, which tries to decode its rows to recover the original information, sent by the source. In order to successfully decode the information, the decoder needs to correctly receive $k$ linearly independent packets.

Figure 1(b) depicts the generalisation of the simple line network described above. Source and destination are separated by $N$ intermediate nodes and the erasures of packets are only at the first channel. Each intermediate node can perform RLNC operations as well.

Another aspect to take into account is the capability of RLNC to reduce the successful decoding probability. Since the code randomly and independently chooses the coefficients of $\mathbf{C}$ over a finite field, the decoding matrix results to be random. Hence, this matrix is not full rank with a probability defined as *failure probability* $P_e$: in particular, its lower and upper bounds are respectively

$$\begin{aligned} P_{e_{lb}} &\propto \frac{1}{q} \\ P_{e_{ub}} &\propto \frac{1}{(q-1)^{n-k}}. \end{aligned} \tag{2}$$

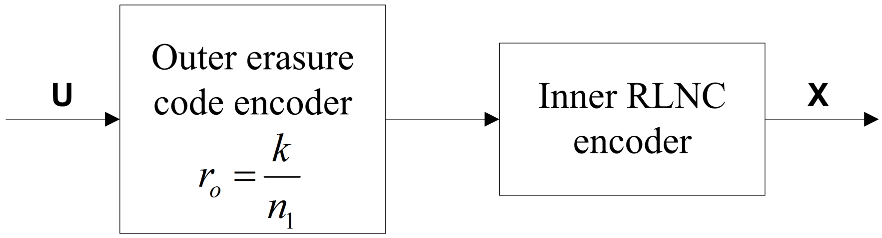


Fig. 2. General encoder structure of a HSCNC. The rates of the outer and inner code are respectively $r_o$ and $r_i$. The $k \times p$ matrix $\mathbf{U}$ and the $n \times p$ matrix $\mathbf{X}$ are respectively the matrices of source and encoded symbols.

TABLE I

COMPLEXITY OF ENCODING AND DECODING OPERATIONS OF RS CODES, LT CODES, RLNC AND HSCNCS. $B$ REPRESENTS THE BATCH SIZE FOR BATS CODES.

| Coding scheme | Encoding complexity | Decoding complexity |
|---|---|---|
| Reed-Solomon | $O\left(\frac{kp(n-k)}{2}\right)$ | $O\left(\frac{kp(n-k)}{2}\right)$ |
| LT | $O\left(np\log(k)\right)$ | $O\left(np\log(k)\right)$ |
| RLNC | $O(np)$ | $O(k^2+kp)$ |
| HSCNC (RS and RLNC) | $O\left(\frac{kp(n-k)}{2}+np\right)$ | $O\left(\frac{kp(n-k)}{2}+k^2+kp\right)$ |
| HSCNC (BATS code) | $O\left(pkB\right)$ | $O\left(kB+pkB\right)$ |

## III. HYBRID SERIAL CONCATENATED NETWORK CODES

In order to improve the performances of RLNC in presence of burst erasures, the article studies the hybrid serial concatenation of RLNC with erasure block codes (i.e. RS codes) and fountain codes (i.e. LT codes). As previously underlined, the erasure code is not applied at link level but only end-to-end. On the other hand, HSCNCs allow linear combination of packets at intermediate nodes.

Figure 2 depicts the general structure of HSCNC encoder. The outer erasure code with rate $r_o = k/n$ encodes $k$ source packets into $n$ encoded packets. Then, the inner RLNEC code introduces spacial redundancy hence, the rate $r_i = 1$. The total rate of the concatenated code becomes $r^{hs} = r_o = k/n$.

Table I lists the complexities of encoding and decoding operations of RS codes, LT codes, RLNC and HSCNCs: LT codes use belief-propagation (BP) decoding algorithm. The complexity of encoding and decoding operations of HSCNCs show that the concatenation of RLNC and RS codes achieves higher complexity than the one achieved by RLNC and LT codes.

## IV. Simulation Results and Discussions

This section presents the performances which can be achieved by network concatenated codes in the scenarios of Figure 1. The simulations were performed in MATLAB. The source collects packets of same size. Next, the outer code encodes the packets and adds redundant information. Afterwards, the RLNC encoder linearly combines the packets. Since RLNC has an intrinsic failure probability, the error decoding probability can sometimes be 1. So, in order to avoid this issue, it was chosen $n_2 = n_1 + 1$: however, the impact of RLNEC code redundancy can be neglected once compared to the one of the inner code.

The burst erasure probability changes in the range $0.005 \leq b_e \leq 0.2$ and burst position in a stream is random. If the line network has more than one channel, the BEC is only the first channel: channels 2 to $N$ are error-free.

Figure 3 depicts the variation of rate of the codes by increasing the burst erasure probability of the first channel. In particular, the coding scheme keeps a failure decoding probability $< 0.01$. First, let consider HSCNCs which have LT codes as outer code. The choice of $m$ to achieve for BATS codes was $m = 2$ for LT code and $m = 8$ for RLNEC code. It is possible to see that BATS codes achieve lower rates than LT codes to recover erasures. Moreover, the presence of intermediate nodes re-encoding packets is not improving the correction capabilities of this HSCNC.

Next, let analyse HSCNCs which have RS codes as outer codes. The size of the finite fields was set to $m_{RS} = m_{RLNC} = 8$. These HSCNCs are more powerful in terms of error-correcting capability. The rate is much higher than the one obtained with BATS codes. Furthermore, the re-encoding operation at the intermediate nodes is significantly improving the erasure correction, especially when the burst erasure probability increases. The re-encoding operations at intermediate nodes improve the error-correcting capability of $\geq 11$ percent for burst erasure probabilities $\geq 0.18$.

In order to verify the higher error correction of RS codes due to re-encoding operations, Figure 4 depicts the comparison of HSCNCs in point-to-point and with twenty intermediate nodes. The results reveals that the novel scheme keeps the performances also when the number of intermediate nodes increases. In this second simulations, $m_{RS} = 9$ and $m_{RLNC} = 8$. Since this time the error-correcting capability increases of 5 percent, it is possible to guess that by augmenting $m_{RS}$, the effect of re-encoding decreases. Nevertheless, in applications, $m_R S$ is often kept low in order not to achieve high complexity of operations.

Figure 5 shows how the encoding and decoding throughput changes by varying the size of the finite field of the codes. In the pairs between brackets, the two values are respectively the $m$ of the erasure/fountain

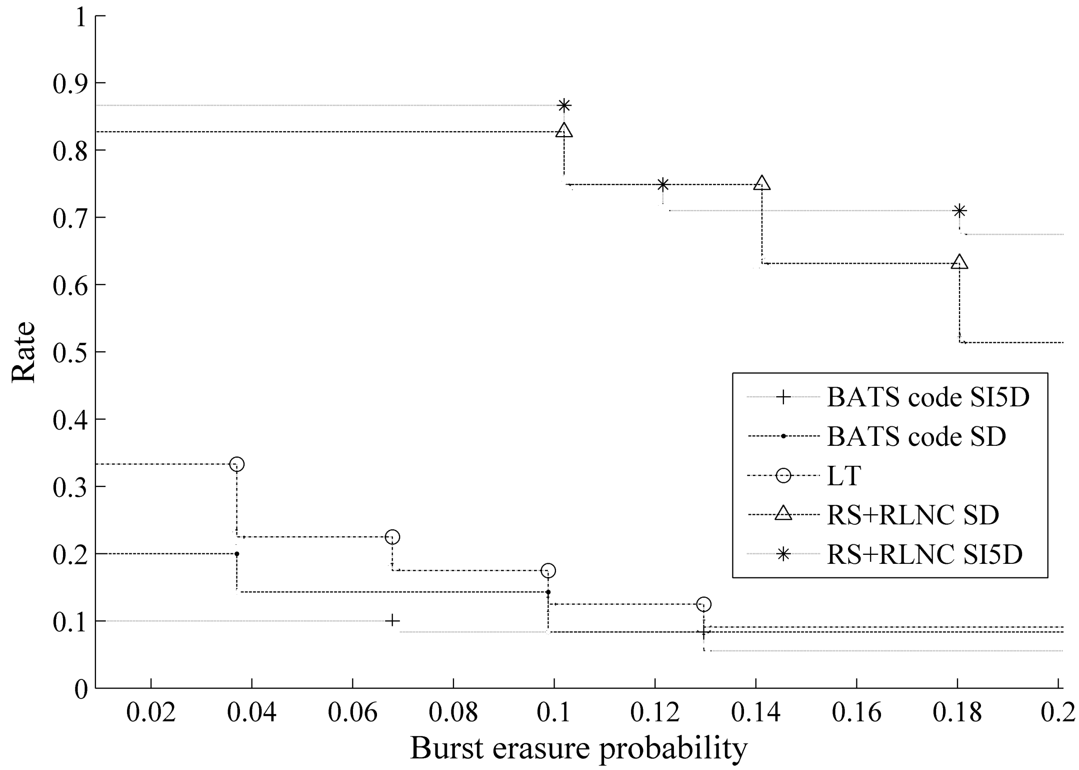


Fig. 3. Rate of LT code, BATS code, HSCNC with RS code and RLNC: the rates of the codes are compared in case of point-to-point communication and line network with five intermediate nodes. The intermediate nodes are re-encoding the packets by performing RLNC. The rates are changed to keep a failure decoding probability $< 0.01$. The codes have $m_{RS} = m_{RLNC} = 8$.

code and the one of the RLNEC code. For the analysis of these results is important to also refer to complexities, listed in Table I.

When RLNC is concatenated with RS codes, the encoding/decoding throughput are almost equal. Since the complexity of RS codes is high, the throughput achieved is low: the variation of $m_{RLNC}$ does not have significant impact on throughput. Furthermore, the optimal values are obtained when $m_{RS} = 8$.

In case of BATS codes, the encoding/decoding throughput is significantly higher. This happens because the complexity of LT codes is lower than RS codes. On the encoding side, the throughput increases once $m_{LT}$ augments. On the decoding side, the complexity of gaussian elimination influences the final throughput: in fact, for $m_{RLNC} = 16$ the BATS code reaches the worst result. Therefore, the choice of $m_{LT}$ has not significant impact on the throughput.

Finally, it is possible to see that HSCNCs that use outer RS codes achieve higher correction capabilities for higher coding rates (i.e. for lower redundancy). Side by side, the price to pay is higher complexity

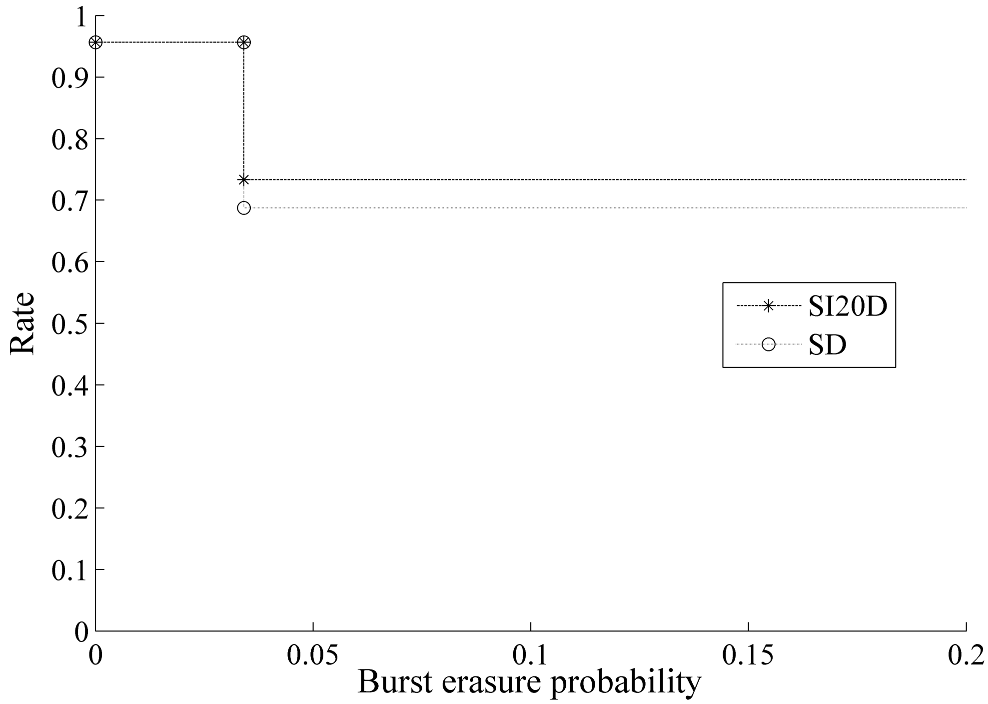


Fig. 4. Rate HSCNC with RS code and RLNC: the rates of the codes are compared in case of point-to-point communication and line network with twenty intermediate nodes. The intermediate nodes are re-encoding the packets by performing RLNC. The rates are changed to keep a failure decoding probability $< 0.01$. The codes have $m_{RS} = 9$ and $m_{RLNC} = 8$.

of encoding/decoding operations. On the other hand, BATS codes need lower coding rates to recover the same number of erasures of HSCNCs with RS codes. However, the important improvement is obtained in terms of encoding/decoding throughput, since the complexity of the coding operations is much lower.

## V. Conclusions

This article proposes the use of HSCNCs to improve the performance of line networks. Given the encoder structure, the proposed approach is also suitable to improve other single source unicast networks. The scope was the optimisation of the error-correcting capability and of the complexity of the codes. The proposed approach was evaluated by MATLAB simulations and theoretical analysis. The outcome of this work mainly reveals that HSCNCs with RS outer codes have higher error-correcting capability than BATS codes. Side by side, BATS codes have higher encoding/decoding throughput and lower complexity than HSCNCs that use RS outer codes. So, HSCNCs with RS outer codes are preferable in scenarios

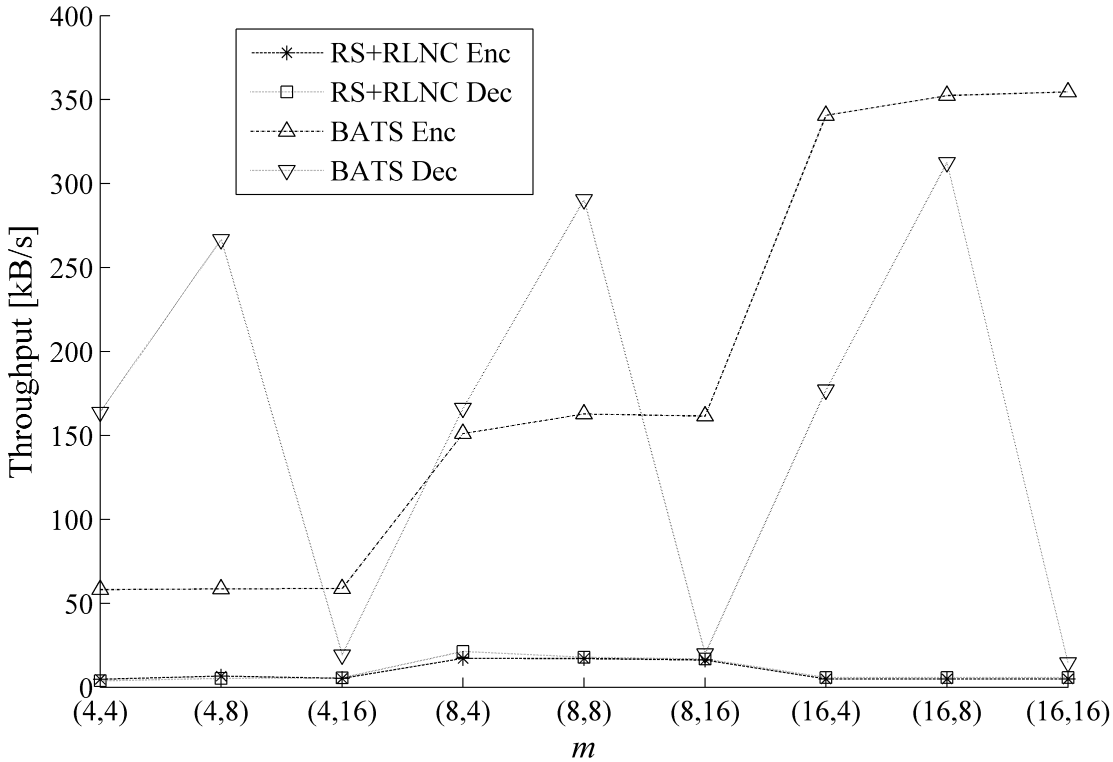


Fig. 5. Encoding and decoding throughput of HSCNCs. On the horizontal axis the pairs are $(m_{outer}, m_{RLNC})$. On the vertical axis, in order to calculate the decoding throughput, only the correct information obtained by the receiver has been used.

where the principal objective is the reliability. On the other hand, BATS codes are more suitable for networks, in which low complexity of operations at the nodes is needed.

## ACKNOWLEDGEMENT

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