Product Network Codes for Reliable Communications in Diamond Networks

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Abstract

In wireless networks, mobile users connect either with other devices or with base stations. They can experience high errors caused by losses, low levels of signals or disconnections. Due to these aspects, it is important to find ways to make the communication reliable. Product network codes represent a way to improve error-correction capability. The main idea is to use a powerful error correction code in time with random linear network coding in space domain. This paper analyses the error-correcting capabilities of product network codes composed by either Luby transform (LT) codes or Reed-Solomon (RS) codes, and RLNC. The kind of errors are burst errors. The results quantify how product network codes improve reliability in case of high burst error probability.

Index Terms

Random linear network coding, Reed-Solomon codes, LT codes, burst errors.

I. Introduction

In current mobile networks, wireless connections can experience high errors. These errors can be caused by losses, low levels of signals or disconnections: these events generate errors in a burst. Burst errors are significantly responsible for low quality of the communications and high energy consumption because of retransmissions. Due to these aspects, it is important to find efficient ways to reduce the impact of these errors on the communication.

During last years, network coding [1] represented a novel way to achieve higher capacity and higher error correction. In 2011, product network codes were proposed by [2]. The main idea is to encode source

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messages by adding redundant symbols with a systematic error-correcting code with powerful error-detection capability. The systematic code is applied to protect the link layer transmissions. Firthermore, the encoded information is also given to random linear network coding (RLNC) encoder which is used to protect the whole multicast (network layer transmission).

This work analyses the error-correcting capabilities of product network codes composed by either Luby transform (LT) codes or Reed-Solomon (RS) codes, and RLNC. In this paper we proposed a different approach: the encoding and decoding operations of the link-layer codes are only performed at source and destination. The intermediate nodes can only re-code the messages with RLNC. The kind of errors considered are burst errors: in particular, the source faces very high error probability. This allows the evaluation of the codes in very unreliable scenarios as well.

The paper is organized as follows. Section II briefly presents product network codes. Section III provides some preliminaries to clarify the achieved results, describes the model used in the rest of the work and how the system is implemented for the simulations. Finally, Section IV shows the simulation results of the product network coding schemes.

II. PRODUCT NETWORK CODES

Product codes are codes that use NEC in the space domain, and the classical error correction code in the time domain. In this paper we implement a method slightly different from the one used by the unpublished work mentioned in [2].

Here, the source message is protected by encoding it with an error-correcting code and then is mapped into source packets. Next, the packets are given to RLNC encoder and then linearly combined. These product codes can guarantee higher error-correcting capabilities than only a 'classical' erasure code or a random linear network code in case of burst erasures.

The product codes investigated in this work are the ones constituted by either LT or RS codes in time domain, and RLNC in space domain. The encoder and decoder of both error-correcting code and RLNC are placed end-to-end, respectively at source and destination. The intermediate nodes are only able to re-code packets by using RLNC. The time-domain code has a rate $R = k/n$. After time-domain encoding, RLNC encoder linearly combines the packets constituted by the n encoded symbols. The sink receives the messages and decodes them first with Gaussian elimination and then with BP decoding.

III. SYSTEM MODEL

Figure 1 represents the diamond scenario studied in this work. The outgoing links of the source are very unreliable and their packets are subject to high burst errors. The intermediate nodes (Node 1 and

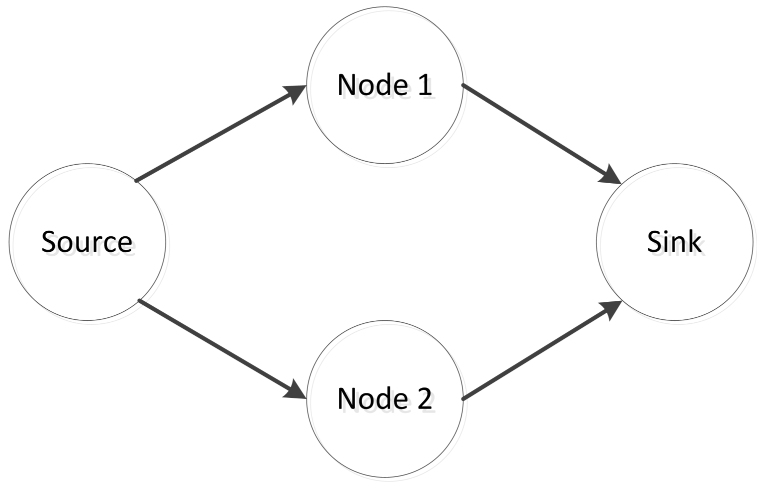

Fig. 1. Diamond scenario, in which a source communicates with a sink via two intermediate nodes. The source transmits encoded packets on both outgoing links: these links are the ones that experience burst errors.

2) re-code the messages received via RLNC. The investigated product network codes are constituted by either RS codes or LT codes and RLNC.

RS codes [3] are non-binary erasure codes with symbols belonging to a Galois field $GF(q)$, with $q = 2^m$. RS codes are very powerful codes for burst error correction. The complexity of RS codes increases with the redundancy: so it is important to pay attention to the trade-off between error-correcting capability and complexity of operations. Because of that, the most interesting RS codes are the ones with high code rates and with small m.

LT codes [4] are a practical implementation of fountain codes. The encoder can generate a variable quantity of encoded symbols according to the needs. The encoder receives a stream of L source bits which are partitioned into $k = L/m$ input symbols over $GF(q)$. The encoder creates symbols according to the so called Robust Soliton distribution. In particular, this probability distribution is the sum between the distribution

$$\tau(i) = \begin{cases} S/ik \quad \text{for } i = 1, \ldots, k/S - 1 \\ S \log\left(S/\delta\right)/k \quad \text{for } i = k/S \\ 0 \quad \text{for } i = k/S + 1, \ldots, k \end{cases} \tag{1}$$

and the ideal Soliton distribution

$$\begin{array}{c} \rho(1) = 1/k \\ \rho(i) = 1/i(i-1) \quad \text{for } i = 2, \ldots, k \end{array} . \tag{2}$$

Hence, after normalising, it results to be

$$\begin{array}{c} \beta = \sum_{i=1}^{k} \rho(i) + \tau(i) \\ \mu(i) = {}^{(\rho(i) + \tau(i))}/_{\beta} \quad \text{for } i = 1, \ldots, k \end{array} . \tag{3}$$

The parameter δ represents the upper bound on the failure probability at the decoder, given a set of n encoded symbols. Next, the variable S is defined as $c\sqrt{k} \log \left({}^{k}/_{\delta} \right)$, where $c > 0$ is a constant. The decoder that is used in this paper is a belief propagation (BP) decoder.

Next, RLNC is a network code that generates random linear combinations of the input packets. The output is a matrix $\mathbf{Y}$ of n_{RLNC} encoded rows. The structure of $\mathbf{Y}$ is

$$\mathbf{Y} = \left[\begin{array}{cc} \mathbf{C} & \tilde{\mathbf{Y}} \end{array} \right] \tag{4}$$

where $\mathbf{C}$ is the matrix of random coefficients of the linear combinations and $\tilde{\mathbf{Y}}$ is the matrix of the codewords. Since the RLNC randomly and independently chooses the coefficients of $\mathbf{C}$ over a finite field, the decoding matrix is random. This matrix may not be full rank with at least a probability

$$P_e \propto \frac{1}{q}. \tag{5}$$

The burst erasure probability is defined as b_e. Each outgoing link of the source has its b_e that is grater than or equal to 0.1.

IV. Analysis and Simulation Results

The first simulated scenario uses a product code of LT codes and RLNC. The source has blocks of information of 1.5 kb. The LT encoder has $\delta = 0.02$. It maps binary data in a finite field with $m_{LT} = 8$. Next, it encodes the k source symbols in $GF(q)$ into n_{LT} symbols according to rate $R = k/n_{LT}$. The packets which contain these n_{LT} symbols are passed to RLNC encoder that linearly combines them. The coefficients of the linear combinations and the coded packets are sent on both outgoing links with fixed burst erasure probabilities. The intermediate nodes re-encode the ingoing packets and transmit their linear combinations on the outgoing links. Finally, the sink collects k linearly independent encoded packets from the two ingoing links and decodes them first with Gaussian elimination and then with BP decoder. Figure 2 depicts the results of the simulations: in particular, the simulations consider high burst error probabilities (> 0.1). The errors are only at the outgoing links of the source.

The second scenario uses RS codes and RLNC. The size of source information is as above. The size of RS finite field is $m_{RS} = 8$. After RS encoding operations, RLNC encodes the packets and sends them on the outgoing links.

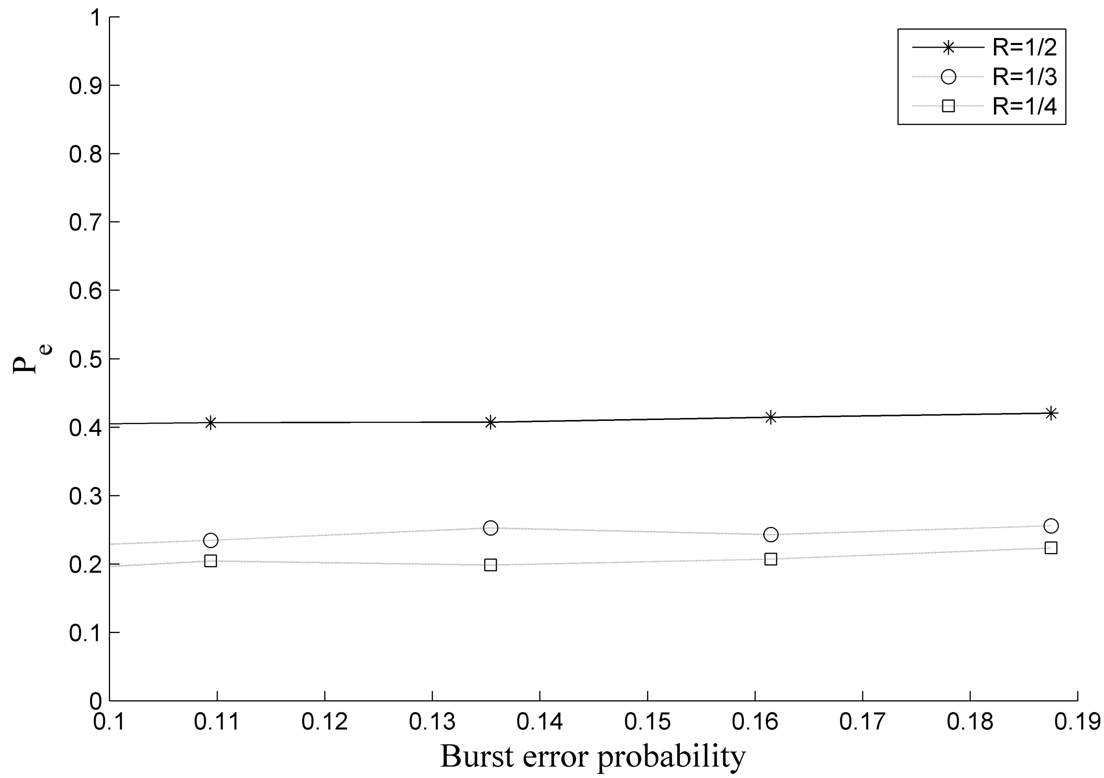

Fig. 2. Decoding error probability for product network codes in diamond scenario. The codes are constituted by LT codes and RLNC. The burst error probability b_e of the horizontal axis is the one of each one of the outgoing links.

By comparing Figure 2 and 3 it is possible to see the behaviour of the two schemes. The performances of the codes are evaluated in terms of error decoding probability (P_e). Product codes that use RS codes have higher error-correcting capabilities than the ones that use LT codes. On the other hand, the complexity of LT codes is lower than the one of RS codes. Furthermore, the performance of product network codes is not very influenced by the increase of burst error probability in both cases.

V. Conclusion

This paper quantified the good performance of product network codes in presence of high burst error probability. The error decoding probability of product codes is slightly influenced by the increase of errors. Next, it was presented that the use of RS codes allows to achieve lower P_e with higher coding rates (i.e. lower redundancy). However, if low complexity is required instead of very high reliability, the use of product codes that combine LT codes and RLNC is suggested.

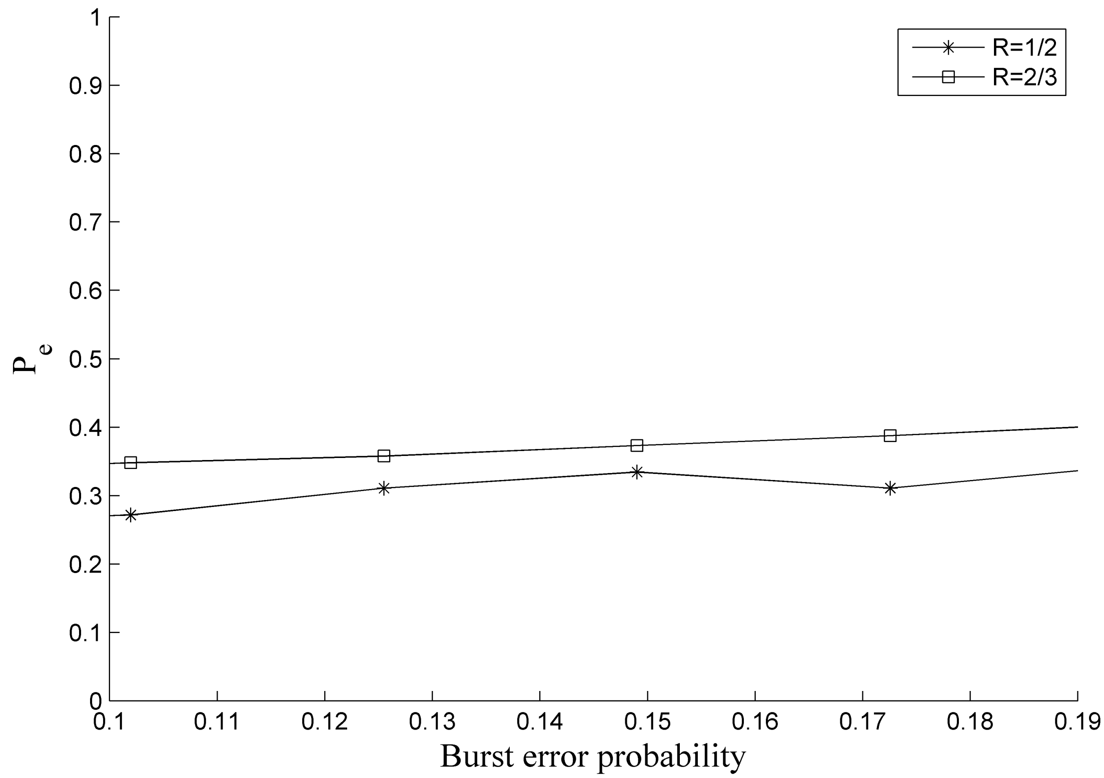

Fig. 3. Decoding error probability for product network codes in diamond scenario. This codes are constituted by RS codes and RLNC. The burst error probability b_e of the horizontal axis is the one of each one of the outgoing links.

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