TECHNISCHE UNIVERSITÄT DRESDEN

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Stochastic Models for Networks and Wireless Communications

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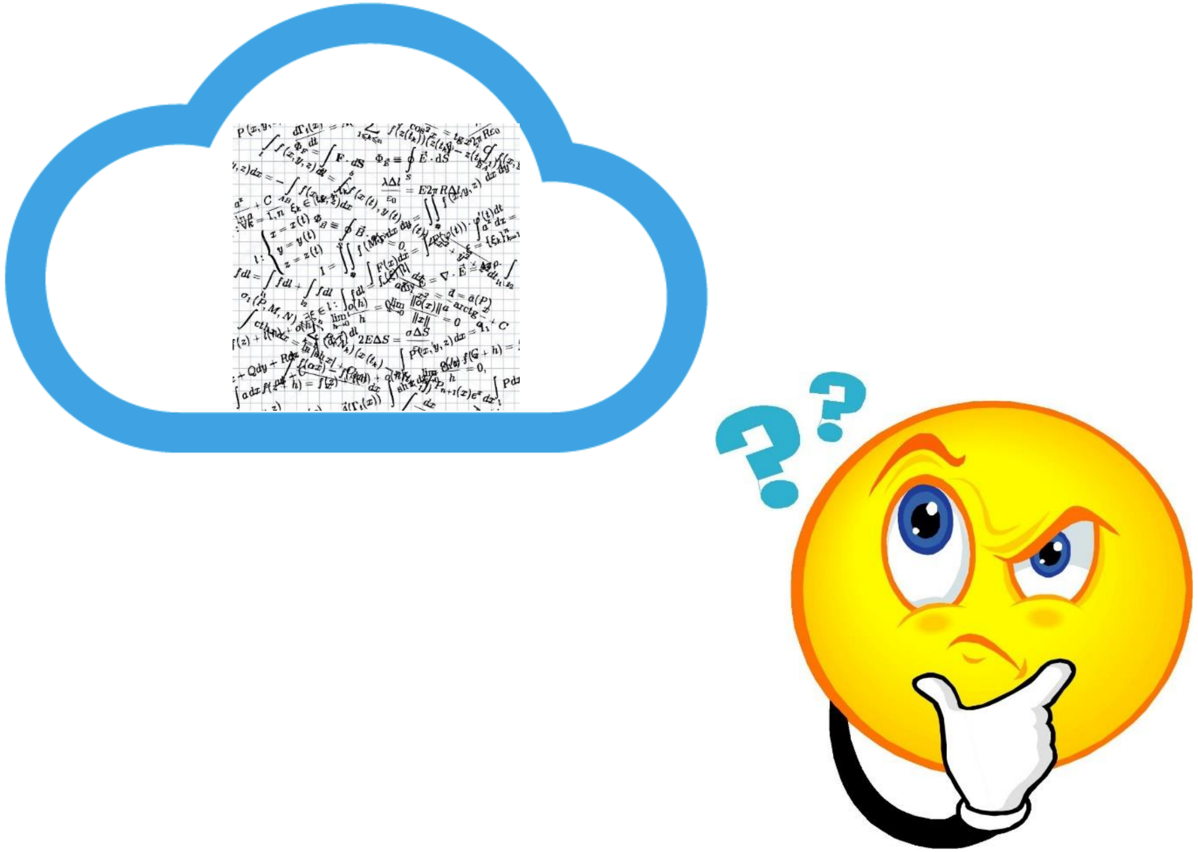
H2020 ETN SECRET Training Event

Stockholm, 11.02.2020

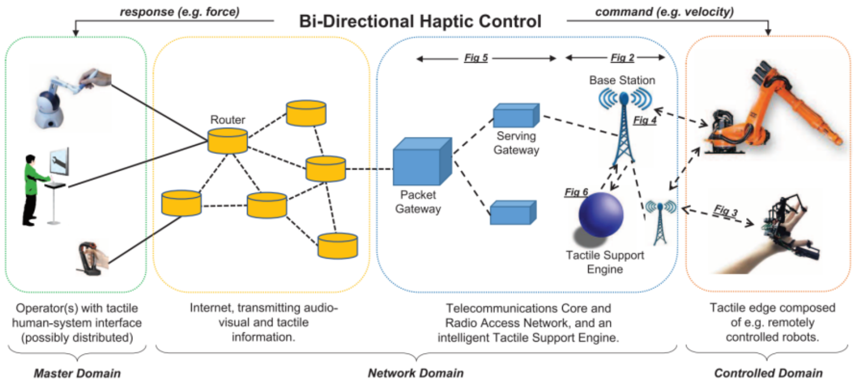
Why Do We Need Theoretical Modelling of Networks?

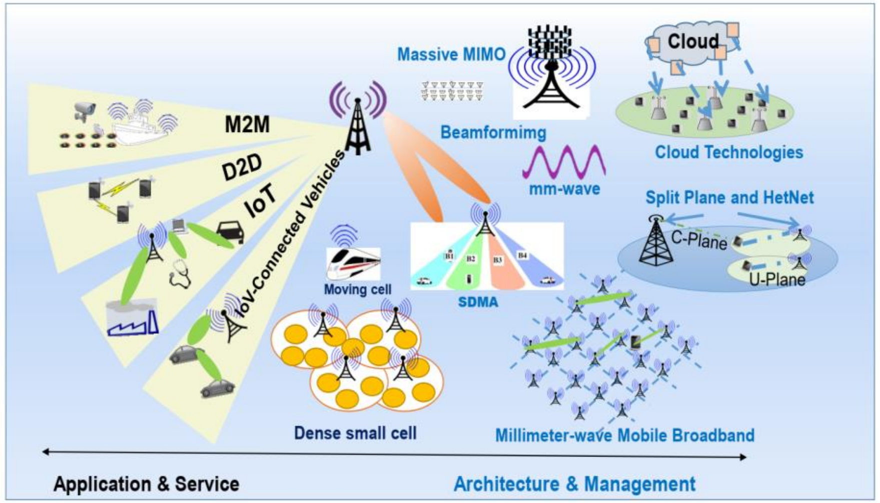


Future Generation Networks

Some reasons why theoretical modelling of future generation networks is necessary:

- Complexity
- Size
- Heterogeneity
- Time-dependent variability
- Cost



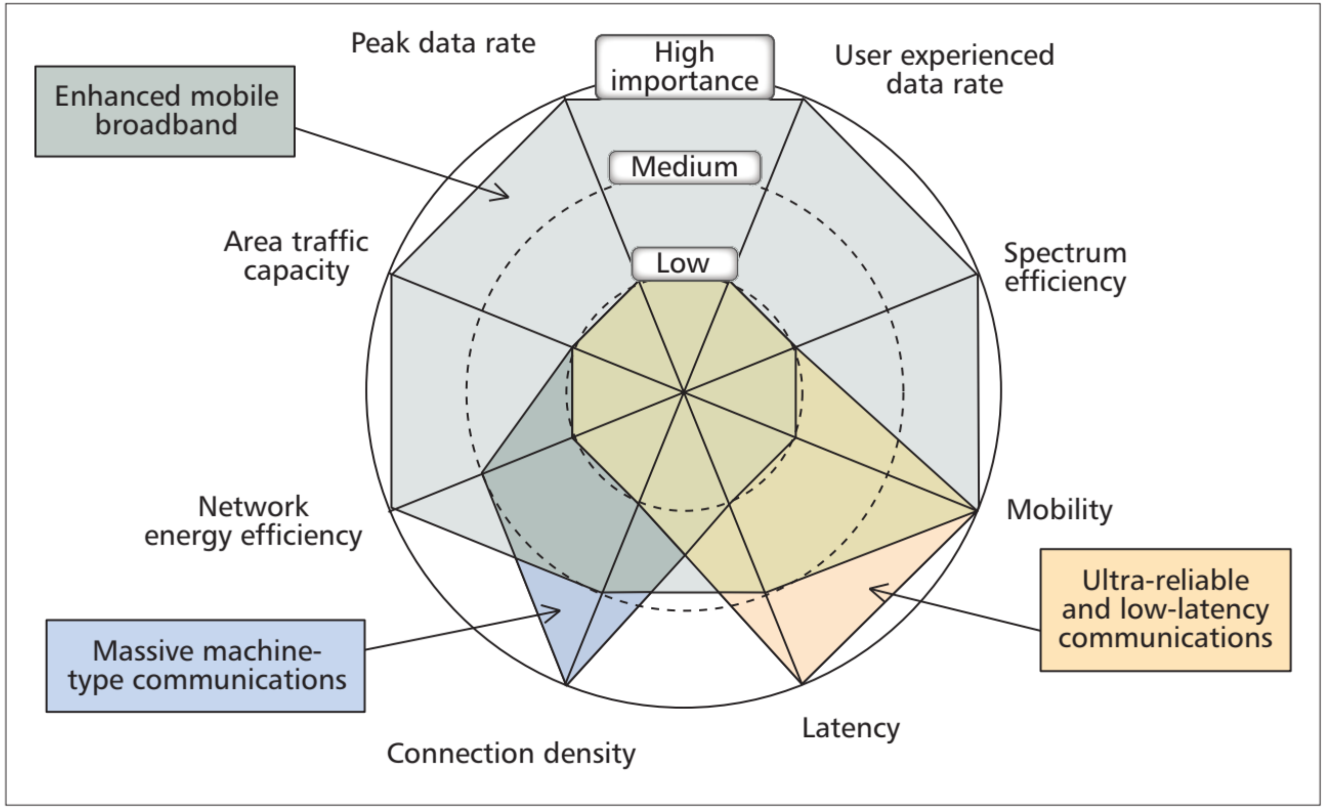


M. Agiwal, A. Roy and N. Saxena, "Next Generation 5G Wireless Networks: A Comprehensive Survey," in *IEEE Communications Surveys & Tutorials*, vol. 18, no. 3, pp. 1617-1655, thirdquarter 2016.

M. Simsek, A. Aijaz, M. Dohler, J. Sachs and G. Fettweis, "5G-Enabled Tactile Internet," in *IEEE Journal on Selected Areas in Communications*, vol. 34, no. 3, pp. 460-473, March 2016.

5G and Beyond Key Performance Indicators (KPIs)

Some KPIs require performance analysis of entire end-to-end communication since each area of the network gives contribution (e.g. latency and reliability)



3G/4G Radio Access Network (RAN)

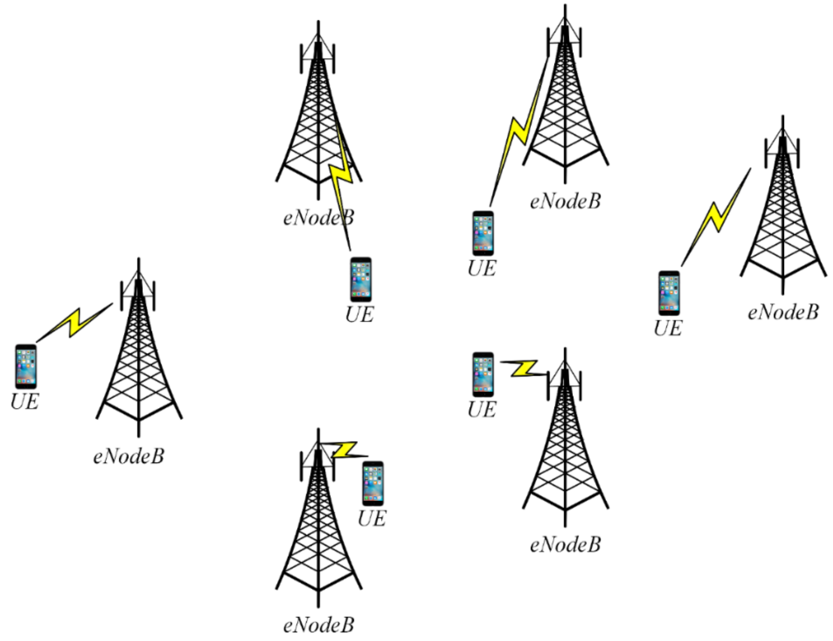
Reliability and maximum throughput at the RAN are mainly affected by:

- Signal-to-noise ratio (SNR)
- Signal-to-interference-plus-noise ratio (SINR)

$$SINR = \frac{S}{W + I} \qquad I = \sum_{i \in T} P_i h_i l_i \qquad l_i = k_0 \| x_i \|^{-\alpha}$$

- Signal-to-interference ratio (SIR): in cellular networks and dense ad hoc networks, noise can be negligible without any appreciable loss of accuracy.

Path loss l_i and interference I depends on spatial distribution of users and base stations/access points (network geometry).



Why Do We Need Stochastic Geometry?

Cellular networks already exist and base stations have already been deployed.

Stochastic geometry gives a general analytical model, which applies on average for all networks' instances in different geographical places. That avoids repeating measurements for each instance of the networks.

Locations of base stations at different geographical locations form random patterns.

Why Do We Need Stochastic Processes?

The mathematical theory of stochastic processes regards the instantaneous state of the system in question as a point of a certain phase space R (the space of states), so that the stochastic process is a function $X(t)$ of the time t, with values in R. It is usually assumed that R is a vector space, the most studied case (and the most important one for applications) being the narrower one where the points of R are given by one or more numerical parameters (a generalised coordinate system). In the narrow case a stochastic process can be regarded either simply as a numerical function $X(t)$ of time taking various values depending on chance (i.e. admitting various realisations $x(t)$ a one-dimensional stochastic process).

Stochastic process. *Encyclopedia of Mathematics.* URL: http://www.encyclopediaofmath.org/index.php?title=Stochastic_process&oldid=26955

Why Do We Need Stochastic Geometry?

- Interference is function of network geometry, which also affects path loss and fading.
- Large wireless networks are characterised by very high uncertainty, which far exceeds point-to-point systems.
- Stochastic geometry allows to study the average behaviour over many spatial realisations of a networks whose nodes are placed according to some probability distribution.
- Stochastic geometry averages over all network topologies seen from a generic node weighted by their probability of occurrence. In stochastic geometry analysis, the network is abstracted to a convenient point process (PP), which captures the network properties. That is, according to the network type, as well as the behaviour of medium access control (MAC) layer, a matching PP is selected to model the positions of network entities.
- Stochastic geometry has been efficiently applied to cellular systems, ultra-wideband, cognitive radio, femtocells, relay networks and it is very useful to characterise ad hoc networks.

Why Do We Need Stochastic Geometry?

Calculation of performance of randomly selected mobile user or average performance of all users.

Some spatially averaged performance metrics stochastic geometry can calculate:

- Outage probability
- Ergodic capacity (measures long-term achievable rate averaged over all channel and interference states)
- Symbol error probability
- Bit error probability
- Pairwise error probability
- Handover rate

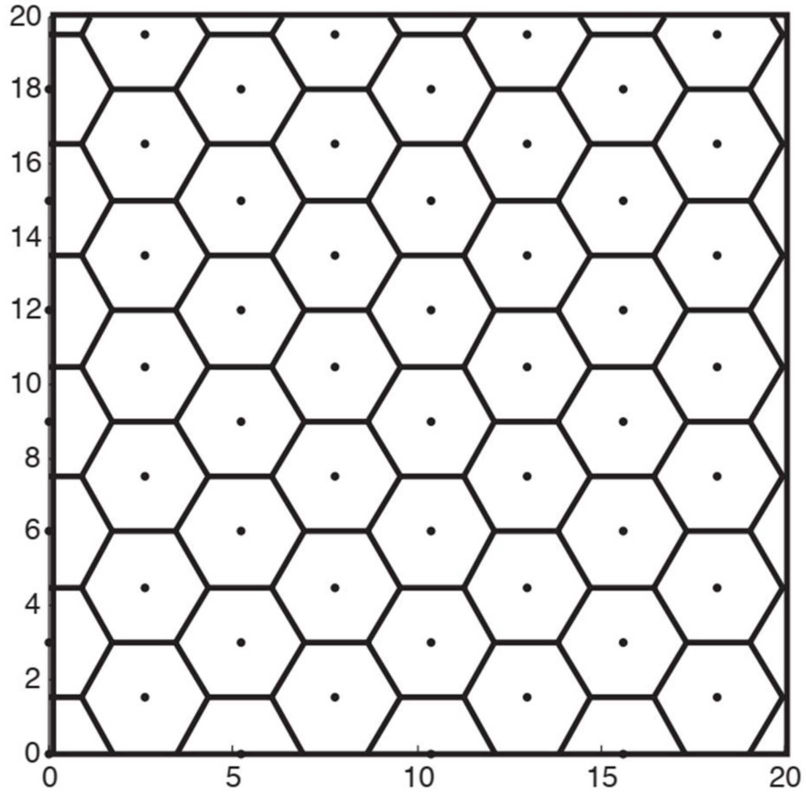
The Beginning of the Story

In the past, cellular networks were represented via the grid-based model:

- Base stations follow deterministic grid, covering hexagonal cells.
- Cells have the same coverage areas.

Drawbacks:

- Accuracy of coverage and interference is disputable.
- Real placement of base stations significantly deviates from this ideal model.



H. ElSawy, E. Hossain and M. Haenggi, "Stochastic Geometry for Modeling, Analysis, and Design of Multi-Tier and Cognitive Cellular Wireless Networks: A Survey," in *IEEE Communications Surveys & Tutorials*, vol. 15, no. 3, pp. 996-1019, Third Quarter 2013.

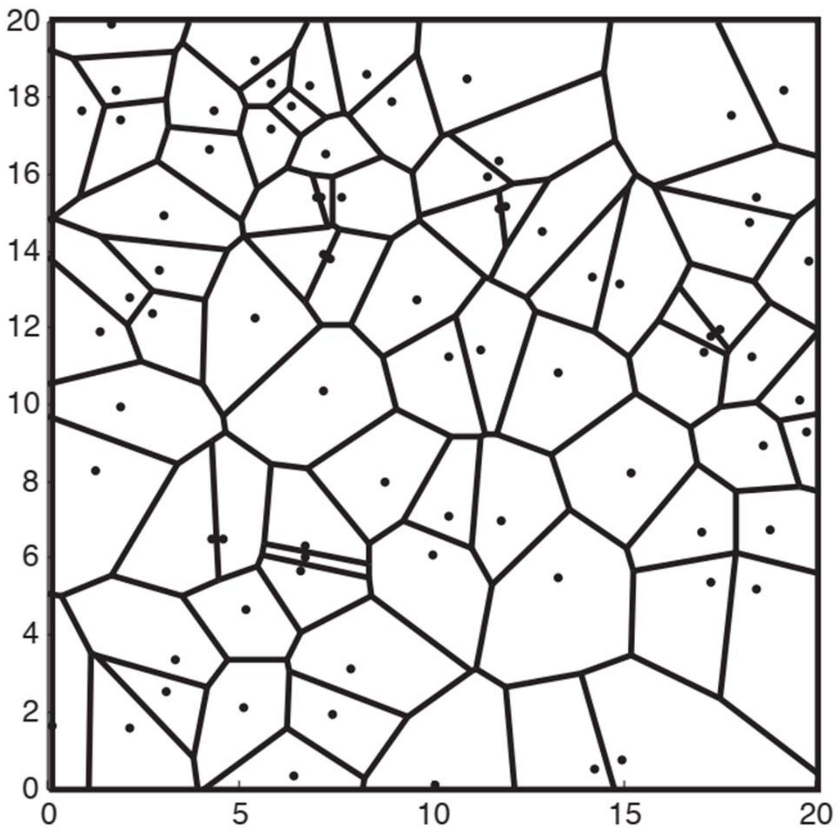
Current Models Based on Random Point Processes

Base stations of cellular networks (e.g. downtown, residential areas, parks, rural areas, etc.) follow random patterns.

By using a Poisson point process (PPP), base stations can be too close to each others since their positions are uncorrelated.

Advantage:

PPP are simple and tractable.



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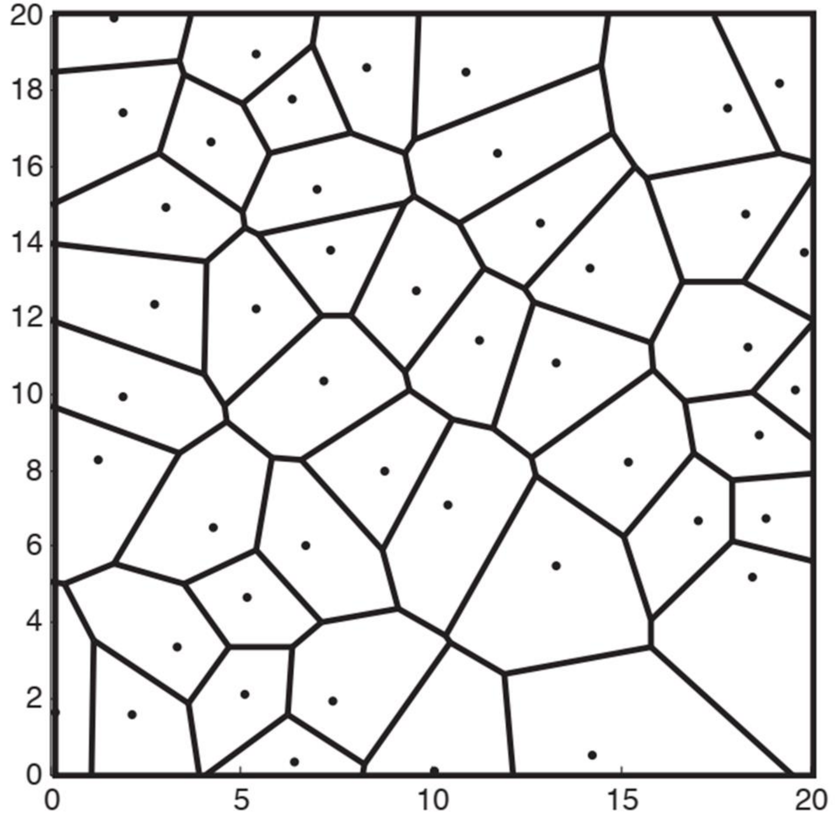
Current Models Based on Random Point Processes

By using repulsive PPs, base stations' pattern reflects more the real placement of base stations provided by an operator.

Matérn hard core point process (HCPP) has a hard core parameter, which reflects the minimum distance between two base stations

Characteristics:

More realistic modelling at the price of analytical tractability.



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Random Point Processes

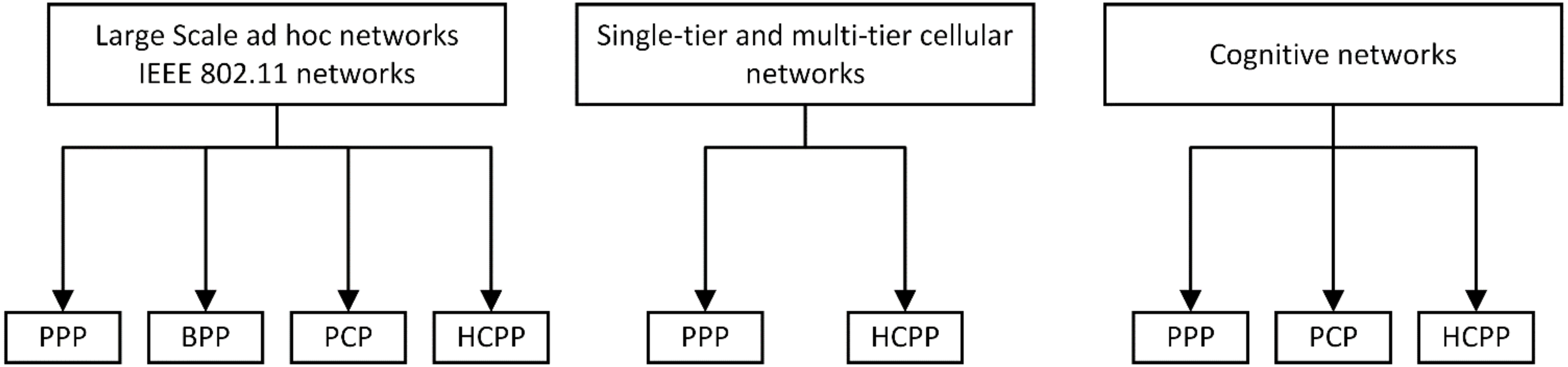
Most Frequent Point Processes for Wireless Networks

Poisson point process (PPP)

Binomial point process (BPP)

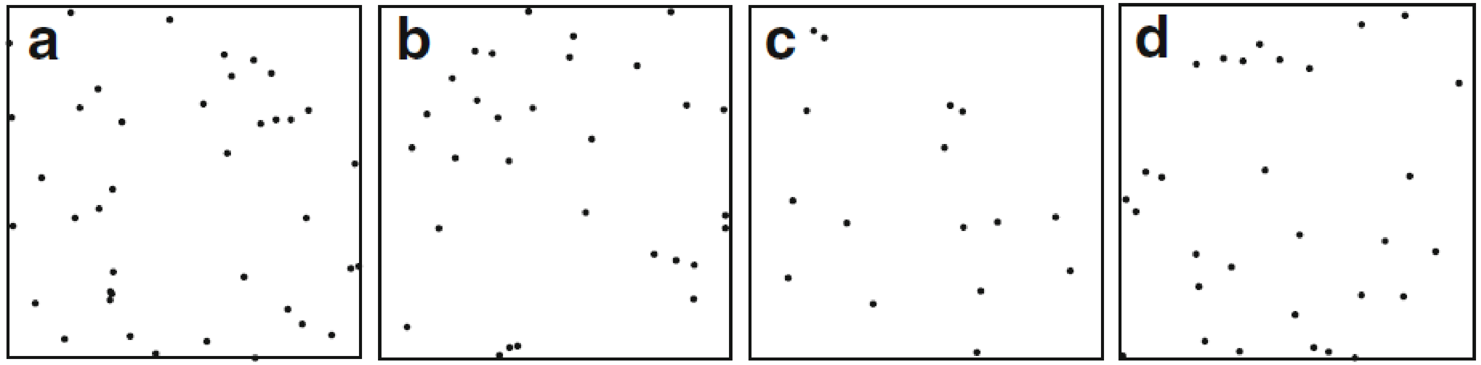
Hard core point process (HCPP)

Poisson cluster process (PCP)



Poisson Point Process

A point process $\Pi = \; x_i, i = 1, 2, \ldots \; \subset \mathbb{R}^d$ is a PPP if and only if the number of points inside any compact set $B \subset \mathbb{R}^d$ is a Poisson random variable, and the numbers of points in disjoint sets are independent.



V. Schmidt, "Stochastic Geometry, Spatial Statistics and Random Fields - Models and Algorithms," Springer 2011.

Poisson Point Process – Modelling a Real Scenario

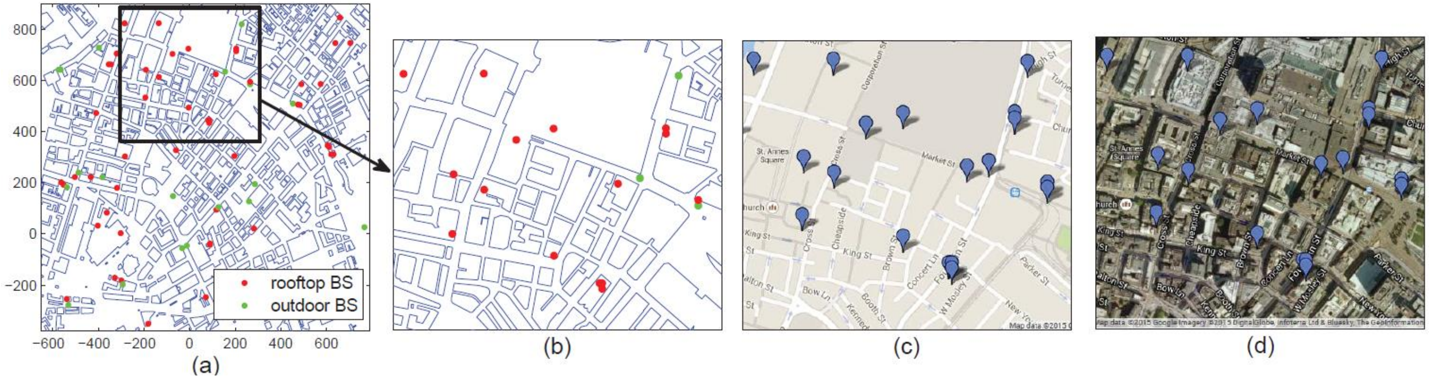
Table 1: BS statistics from OFCOM - The city of London ($\mathcal{A} = 4\,\mathrm{km}^2$).

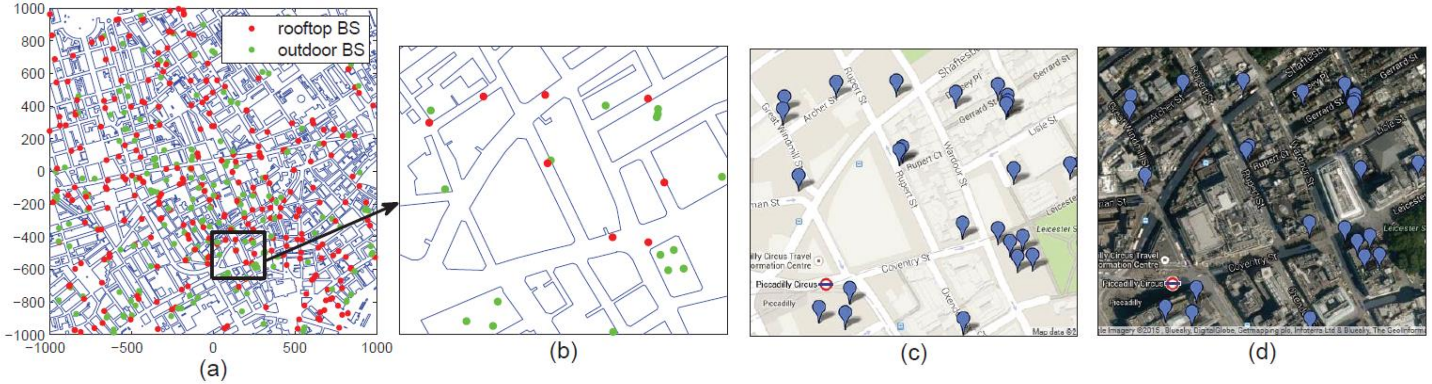
	O2+Vod.	O2	Vod.
Number of BSs	319	183	136
Number of rooftop BSs	95	62	33
Number of outdoor BSs	224	121	103
Average cell radius (m)	63.1771	83.4122	96.7577

Table 2: BS statistics from OFCOM - The city of Manchester ($\mathcal{A} = 1.8\,\mathrm{km}^2$).

	O2+Vod.	O2	Vod.
Number of BSs	37	16	22
Number of rooftop BSs	25	12	14
Number of outdoor BSs	12	4	8
Average cell radius (m)	125.925	191.492	163.305

$$\lambda_{BS} = N / A$$





W. Lu and M. Di Renzo, “Stochastic geometry modeling of cellular networks: Analysis, simulation and experimental validation,” in ACM International Conference on Modeling, Analysis and Simulation of Wireless and Mobile Systems, Cancun, Mexico, Nov. 2015.

Binomial Point Process

The binomial point process models the random patterns produced by a fixed number of points N in a set $B \subset \mathbb{R}^d$ with a finite Lebesgue measure $L(B) < \infty$, where $L(\cdot)$ denoted the Lebesgue measure.

Let $\Pi = \{x_i, i = 1, 2, \ldots\} \subset \mathbb{R}^d$ and $\Pi \subset B$, then Π is a BPP if the number of points inside a compact set $b \subseteq B$ is a binomial random variable, and the numbers of points in disjoint sets are related via multinomial distribution.

Hard Core Point Process

An hard core point process is a repulsive point process where no two points of the process coexist with a separating distance less than a predefined hard core parameter r_h.

A point process $\Pi = \; x_i, i = 1, 2, \ldots \; \subset \mathbb{R}^d$ is an HCPP if and only if $\left\| x_i - x_j \right\| \geq r_h$, $\forall x_i, x_j \in \Pi, i \neq j$, where $r_h \geq 0$ is a predefined hard core parameter.

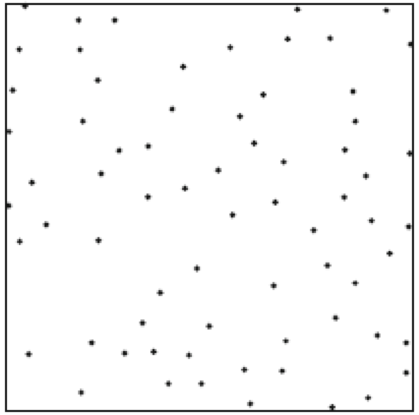
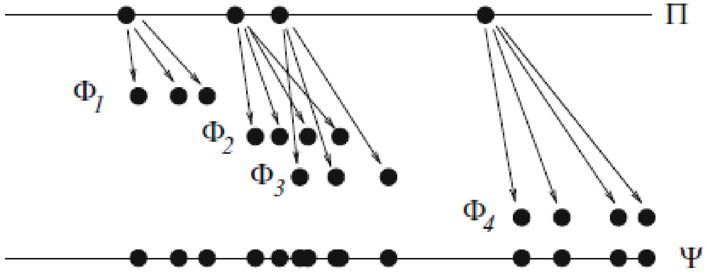


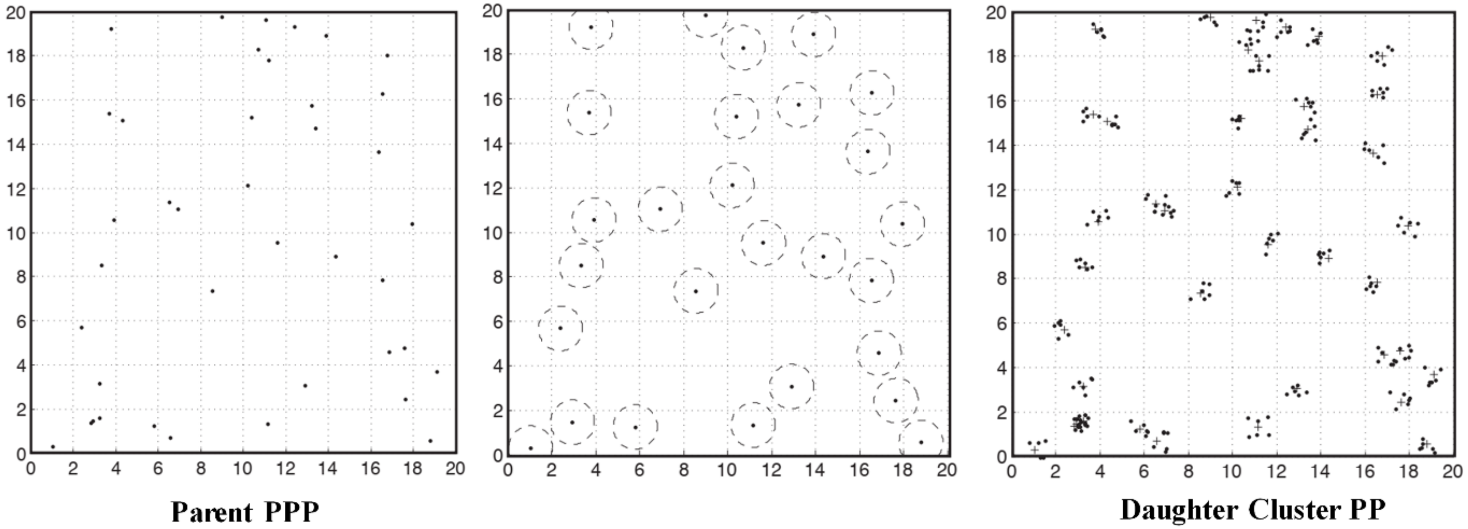
Fig. 3.20 Realization of the hard core process with $\beta = 200$ and $r = 0.07$ in the unit square

V. Schmidt, "Stochastic Geometry, Spatial Statistics and Random Fields - Models and Algorithms," Springer 2011.

Poisson Cluster Process

The Poisson cluster process models the random patterns produced by random clusters. The PCP is constructed from a parent PPP $\Pi = \; x_i, i = 1, 2, \ldots \; \subset \mathbb{R}^d$ by replacing each point $x_i \in \Pi$ with a cluster of points $M_i, \forall x_i \in \Pi$, where the points in M_i are independently and identically distributed in the spatial domain.





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Heavy-tailed Distribution

α-stable distribution is a heavy-tailed distribution.

A random variable X is said to obey to α-stable distribution if there are parameters $0 \le \alpha \le 2, \sigma \ge 0$, $-1 \le \beta \le 1, \mu \in \mathbb{R}$ such that its characteristic function is in the form

$$\phi(\omega) = \mathrm{E}\left[\exp\left(j\omega X\right)\right] = \exp\left\{-\sigma^{\alpha}\left|\omega\right|^{\alpha}\left(1 - j\beta\,\mathrm{sgn}(\omega)\Phi\right) + j\mu\omega\right\}$$

where

$$\Phi = \begin{cases} \tan\dfrac{\pi\alpha}{2}, & \alpha \neq 1 \\ -\dfrac{2}{\pi}\ln\left|\omega\right|, & \alpha = 1 \end{cases}$$

Heavy-tailed Distribution – Modelling a Real Scenario

Information from China Mobile about distribution of base stations in a well-developed eastern province of China. The collected dataset, containing over 47 000 BSs of GSM cellular networks and serving over 40 million

Attributes	City A	City B	City C
No. of BSs	8826	5746	4613
City Area	16,847 km^2	9,816 km^2	9,413 km^2
Population	8.844 million	7.639 million	6.038 million
Description	Inland Provincial Captial	Coastal	Coastal

(a) (b) (c)

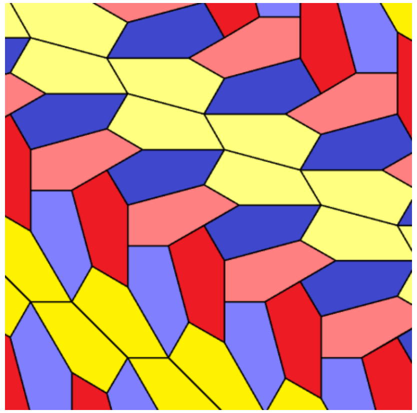
Y. Zhou, R. Li, Z. Zhao, X. Zhou and H. Zhang, "On the α-Stable Distribution of Base Stations in Cellular Networks," in *IEEE Communications Letters*, vol. 19, no. 10, pp. 1750-1753, Oct. 2015.

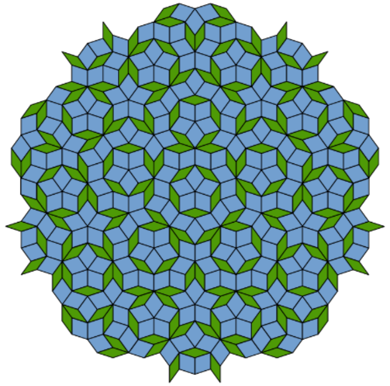
Tessellation Theory

Definition of Tessellation

A <u>tessellation</u> of a flat surface is the tiling of a plane using <u>one or more geometric shapes, called tiles</u>, with no overlaps and no gaps. In mathematics, tessellations can be generalised to higher dimensions and a variety of geometries.

A periodic tiling has a repeating pattern. Some special kinds include regular tiling with regular polygonal tiles all of the same shape, and semiregular tiling with regular tiles of more than one shape and with every corner identically arranged.





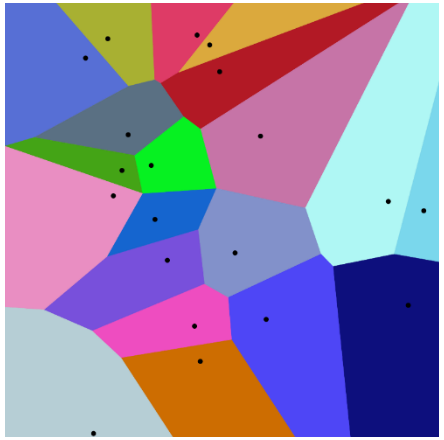
Wikipedia contributors. "Tessellation." *Wikipedia, The Free Encyclopedia*. Wikipedia, The Free Encyclopedia, 4 Feb. 2020. Web. 6 Feb. 2020.

Why Is Tessellation Needed?

Tessellation models the virtual cellular structure of coverage in wireless cellular networks.

Hypothesis: mobile users connect to the closest access point/base station in order to have better radio signal strength (RSS), which is the so called RSS-based association rule.

The kind of tessellation that can reasonably model wireless cellular networks, according to the RSS-based association rule, is called Voronoi tessellation.



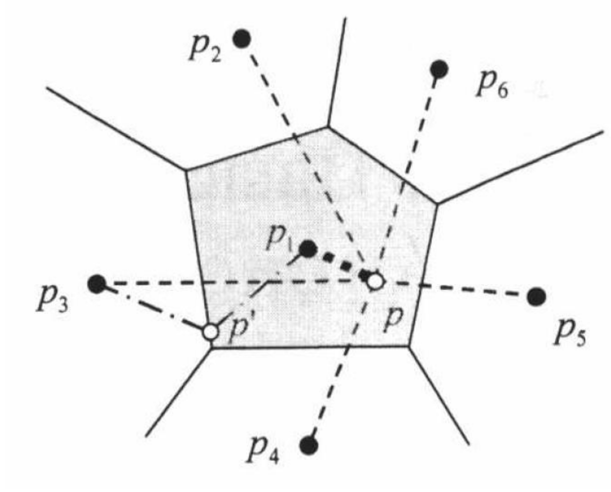
Wikipedia contributors. "Voronoi diagram." *Wikipedia, The Free Encyclopedia*. Wikipedia, The Free Encyclopedia, 29 Dec. 2019. Web. 6 Feb. 2020.

Voronoi Diagrams

The Voronoi diagrams of a set of generators (points) is a collection of regions, which divides the two-dimensional Euclidean space.

Each region corresponds to one of the generators, and all of the points in one region are closer to the corresponding point than to any other one.

Where there is not one closest point, there is a boundary.



A. Dobrin, “A review of properties and variations of voronoi diagrams.” [Online]. Available: https://www.whitman.edu/Documents/Academics/Mathematics/dobrinat.pdf.

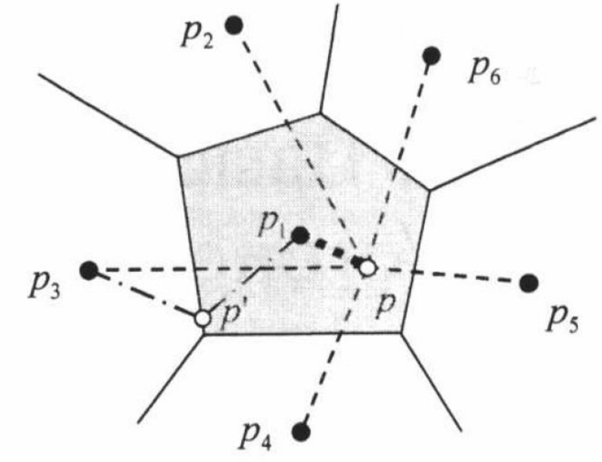
Voronoi Diagrams

Let's denote the location of a point p_i as $(x_{i1}; x_{i2})$, thus the corresponding vector becomes $\mathbf{X}$. Let $P = \ p_1, p_2, \ldots, p_n \ \in \mathbb{R}^2$ where $2 \leq n \leq \infty$ and $p_i \neq p_j, i \neq j$ and $\forall i, j = 1, 2, \ldots, n$ be the set of <u>generator points</u>, or <u>generators</u>. We call the region given by

$$V(p_i) = \ \mathbf{x} \mid \left\| \mathbf{x} - \mathbf{x}_i \right\| \leq \left\| \mathbf{x} - \mathbf{x}_j \right\| \forall j \ni i \neq j$$

the Voronoi region of p_i, where $\|\cdot\|$ is the usual Euclidean distance.

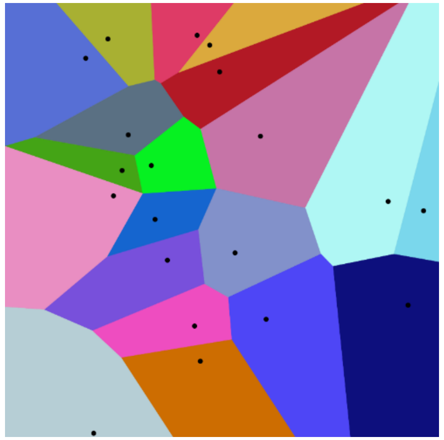
Notation $V(p_i)$ can also be referred to as V_i. All Voronoi regions in an ordinary Voronoi are connected and convex.

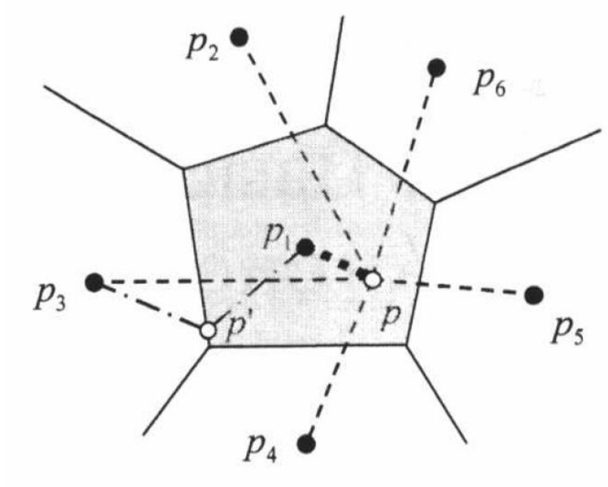


Voronoi Diagrams

We call the set given by $\mathbb{V} = \ V(p_1), V(p_2), \ldots, V(p_n)$, the Voronoi diagram of P.

By using a different set-theoretic notation becomes $\mathbb{V} = \bigcup_{i=1}^{n} V_i$.



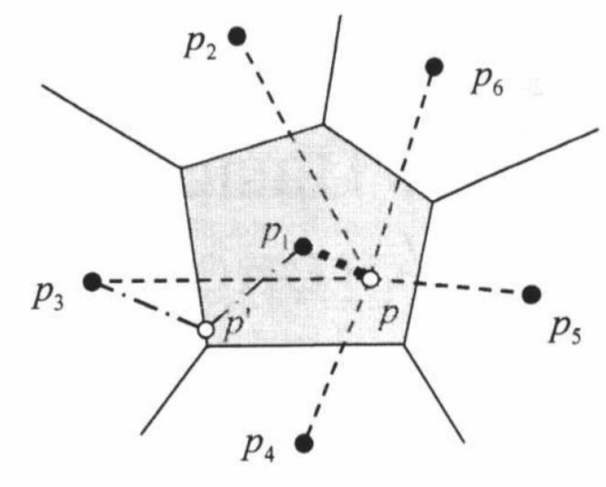


Basic Components of the Voronoi Diagram

The Voronoi diagram is composed of three elements: generators, edges, and vertices.

P is the set of <u>generators</u>.

Every <u>point on the plane</u> that is not a vertex or part of an edge is a point in a distinct Voronoi region.



Basic Components of the Voronoi Diagram

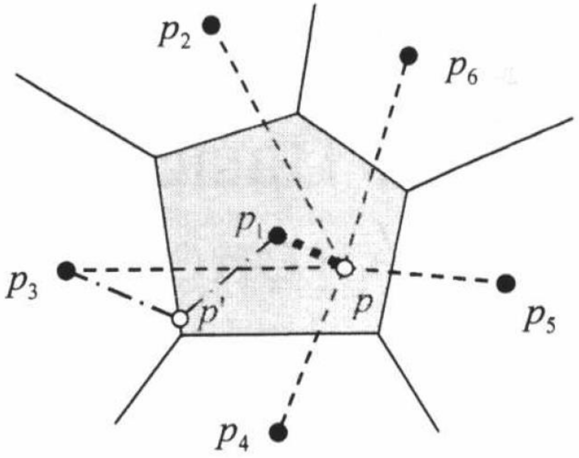
An edge between the Voronoi regions V_i and V_j is $V_i \cap V_j = e(p_i, p_j)$.

If $e(p_i, p_j) \neq 0$, V_i and V_j are considered adjacent.

Any point $\mathbf{X}$ on $e(p_i, p_j)$ has the property that $\|\mathbf{x}-\mathbf{x}_i\| = \|\mathbf{x}-\mathbf{x}_j\|$.

An edge can be denoted as e_i , where i is an index for the edges and does not have to be related to the index of generator points.

The set of edges surrounding a Voronoi region V_i can be referred to as $\partial V(p_i)$ or ∂V_i . Moreover, also the boundary of V_i .

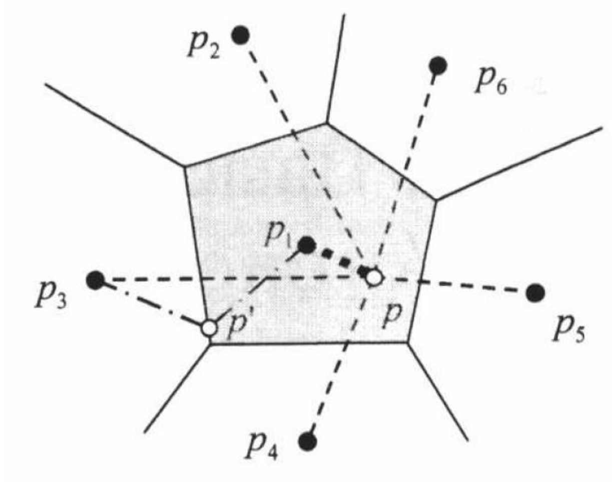


Basic Components of the Voronoi Diagram

A vertex is located at any point that is equidistant from the three (or more) nearest generator points on the plane.

Vertices are denoted q_i , and are the endpoints of edges. The number of edges that meet at a vertex is called the degree of the vertex.

If $\forall q_i \in \mathbb{V},\ \mathrm{degree}(q_i) = 3$, then $\mathbb{V}$ is considered to be non-degenerate. Otherwise, $\mathbb{V}$ is considered degenerate.



Voronoi Diagrams on a Bounded Subset of $\mathbb{R}^2$

While most of the time we will consider Voronoi diagrams on $\mathbb{R}^2$, we can also have them on any set $S \subseteq \mathbb{R}^2$.

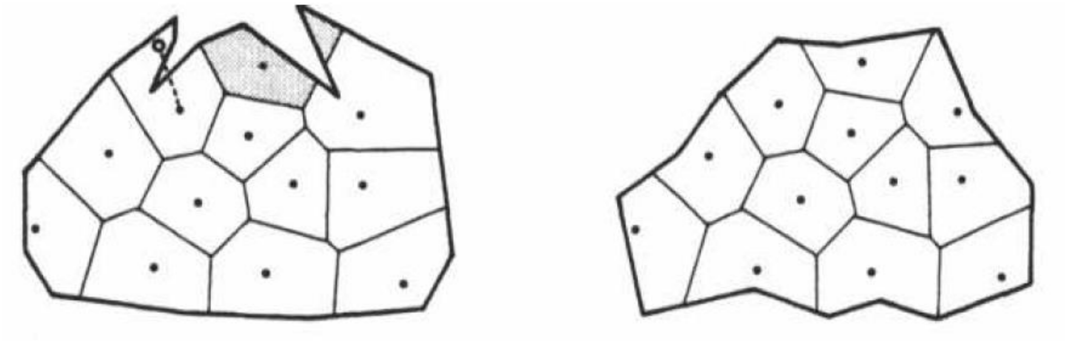
Let's assume S to be non-empty, for a Voronoi diagram on an empty set would be trivial.

The bounded Voronoi diagram is defined by $\mathbb{V} = \; V_1 \cap S, V_2 \cap S, \ldots, V_n \cap S \;$.

If, for any i, V_i shares the boundary of S, we call $V_i \cap S$ a boundary Voronoi region. Unlike ordinary Voronoi regions, boundary Voronoi regions need not be connected or convex.

Voronoi Diagrams on a Bounded Subset of $\mathbb{R}^2$

Two Voronoi diagrams generated by the same set *P*, but on different subsets of $\mathbb{R}^2$. In the left diagram, the shaded region is not connected, and in both diagrams, many of the regions are not convex. Note that the non-convex regions are boundary regions.



A. Dobrin, “A review of properties and variations of voronoi diagrams.” [Online]. Available: https://www.whitman.edu/Documents/Academics/Mathematics/dobrinat.pdf.

Dominance Regions

Given any two generators p_i and p_j, the perpendicular bisector of the line connecting p_i and p_j is

$$b(p_i, p_j) = \ \mathbf{x} \mid \|\mathbf{x} - \mathbf{x}_i\| = \|\mathbf{x} - \mathbf{x}_j\| \ , i \neq j$$

Next, $H(p_i, p_j) = \ \mathbf{x} \mid \|\mathbf{x} - \mathbf{x}_i\| \leq \|\mathbf{x} - \mathbf{x}_j\| \ , i \neq j$ is the dominance region of p_i over p_j, and consists of every point of the plane that is closer to p_i than p_j or equidistant from the two.

Then, $H(p_j, p_i)$ or $Dom(p_j, p_i)$ is the dominance region of p_j over p_i.

In the Voronoi diagram in $\mathbb{R}^2$, $H(p_j, p_i)$ is a half-plane.

Voronoi Diagrams Defined via Dominance Regions

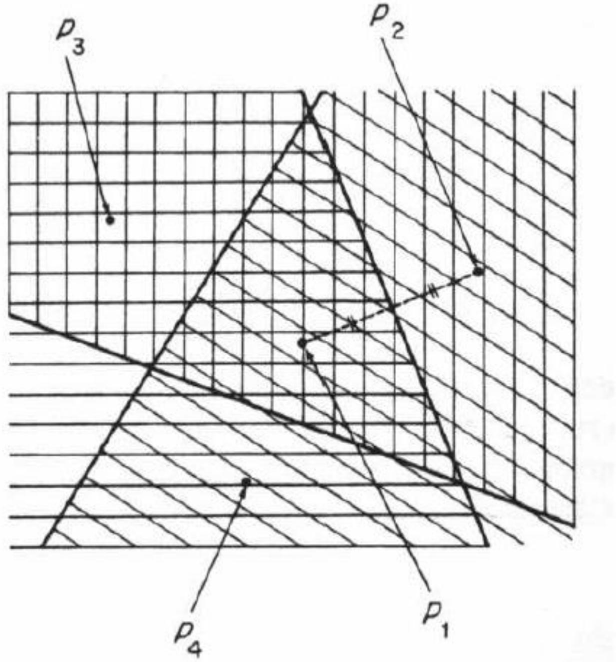
From this definition of dominance regions, it is possible to define Voronoi regions in another way.

Let $P = \; p_1, p_2, \ldots, p_n \; \in \mathbb{R}^2$ be a set of generator points. Let $V_i = \bigcap_{j \in \mathbb{Z}^+ \leq n} H(p_i, p_j)$ be the ordinary Voronoi region associated with p_i .

The set $\mathbb{V} = \; V_1, V_2, \ldots, V_n \;$ is the Voronoi diagram on $\mathbb{R}^2$, generated by P.

Example of construction of a Voronoi region using dominance regions.

By drawing in the half-planes associated with p_1, it is possible to see how a Voronoi region is created by intersecting half-planes.



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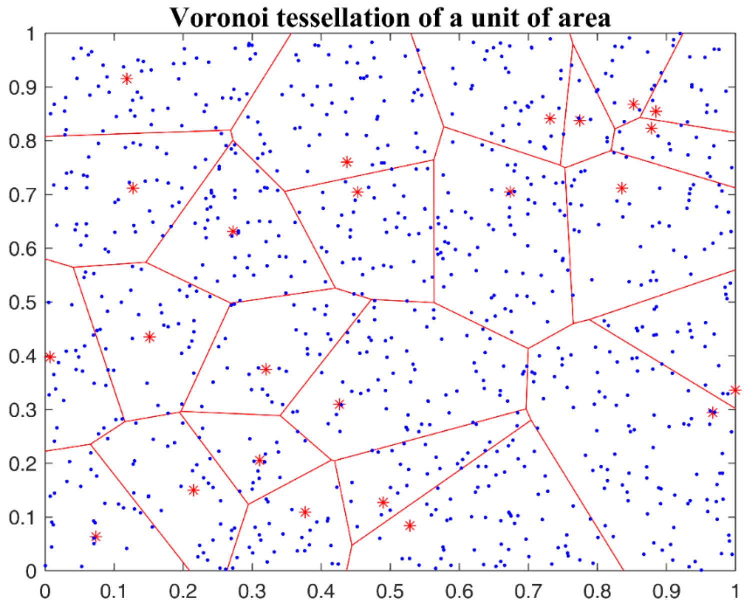
Mobile Users in Single-tier Networks

Given Voronoi tessellation of a cellular network, where base stations and mobile users are distributed according to two independent Poisson point processes, the probability of having N_m mobile users in a cell can be calculated as

$$P\;\; N_m = n \;\; = \frac{3.5^{3.5}\,\Gamma(n+3.5)\left(\lambda_m/\lambda_{BS}\right)^{n}}{\Gamma(3.5)\,n!\left(\lambda_m/\lambda_{BS}+3.5\right)^{n+3.5}}$$



M. D. Renzo, W. Lu, and P. Guan, “The intensity matching approach: A tractable stochastic geometry approximation to system-level analysis of cellular networks,” vol. abs/1604.02683. [Online]. Available: http://arxiv.org/abs/1604.02683

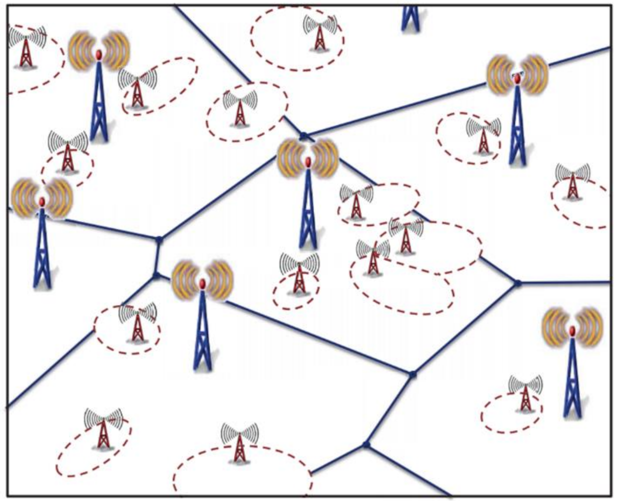
Heterogeneous Networks

Problem Statement

Nevertheless current 4G networks and future generation networks consists of different kind of base stations (i.e. different tiers).

A straightforward unifying model for heterogeneous cellular networks would consist of *K* spatially and spectrally coexisting tiers, where each tier is distinguished by its transmit power, base stations' density and data rate.

Traditional base stations (tier 1) would typically have a much higher transmit power and lower density and offered rate than the lower tiers (e.g. pico and femtocells).



H. S. Dhillon, R. K. Ganti, F. Baccelli, and J. G. Andrews, "Modeling and analysis of k-tier downlink heterogeneous cellular networks," IEEE Journal on Selected Areas in Communications, vol. 30, no. 3, pp. 550–560, Apr. 2012.

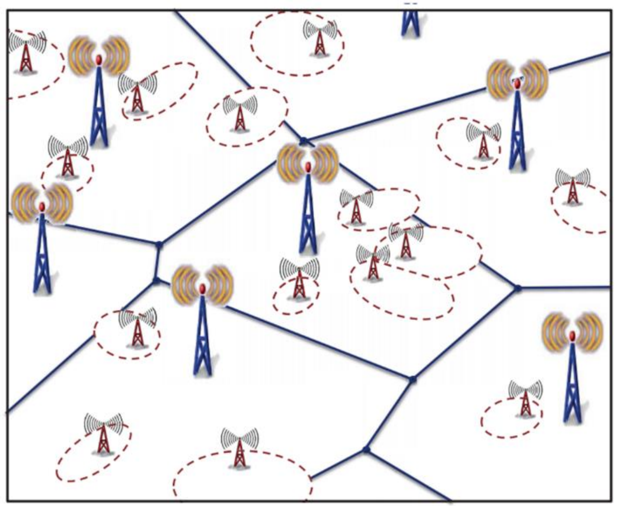
Problem Statement

Let assume the base stations in the *i*-th tier are spatially distributed as a PPP Φ_i of intensity λ_i , transmit power P_i , and have a SINR target of β_i .

A mobile can reliably communicate with a base station in the *i*-th tier iif its downlink SINR with respect to that base station is greater than β_i .

Thus, each tier can be uniquely defined by the tuple P_i, β_i, λ_i .

The mobiles are also modelled by an independent PPP Φ_m of density λ_m.



Coverage Regions in Heterogeneous Networks

The coverage regions can be visually plotted in two steps. First, we randomly place K different types of base stations on a two-dimensional space according to the aforementioned independent PPPs.

Ignoring fading, the space is then fully tessellated following the maximum SINR connectivity model. Note that in reality the cell boundaries are not as well defined as shown in these coverage regions due to fading. Therefore, these plots can be perceived as the average coverage footprints over a period of time so that the effect of fading is averaged out.

Due to the differences in the transmit powers over the tiers, these average coverage plots do not correspond to a standard Voronoi tessellation. Instead, they closely resemble a circular Dirichlet tessellation, also called a multiplicatively-weighted Voronoi diagram.

Coverage Regions and the Effect of Fading

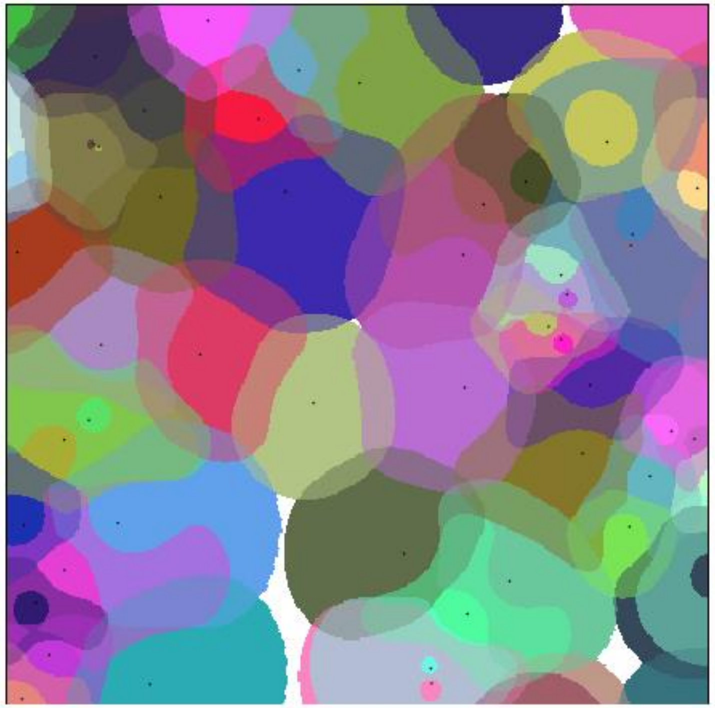


Fig. 7.1 SINR coverage model without fading.

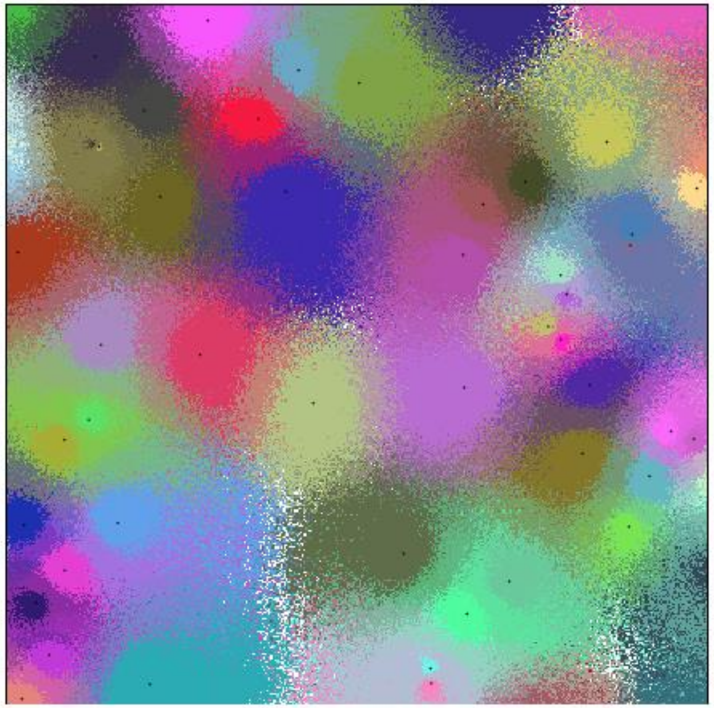


Fig. 7.2 SINR coverage model with point dependent fading.

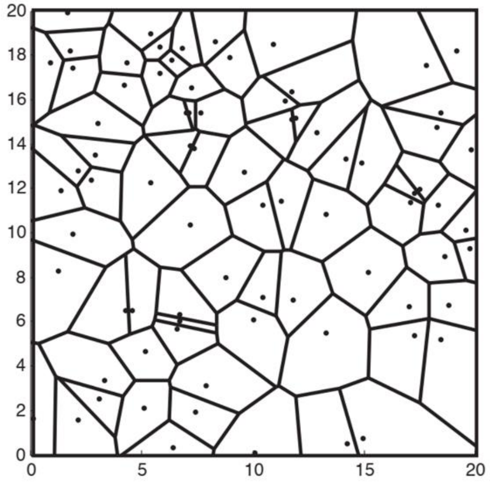
F. Baccelli, B. Blaszczyszyn, “Stochastic Geometry and Wireless Networks, Volume I -Theory,” Foundations and Trends in Networking Vol. 3: No 3-4, pp 249-449,

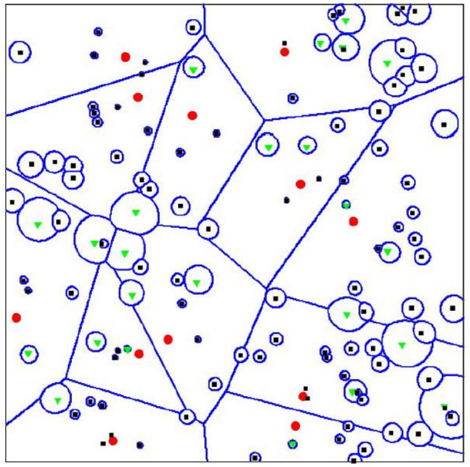
Coverage Regions for a Three-tier Network

The coverage regions are for macro, pico and femto-cell tiers. As in actual networks, the assumption is that macro-cells have the highest and the femto-cells have the lowest transmit power, with pico-cells somewhere in between.

For example, in LTE, typical values are on the order of 50W, .2W, and 2W, respectively. Therefore, femto-cell coverage regions are usually much smaller than the other two tiers, particularly when they are nearby a higher power base station.

Similarly, the coverage footprint of pico-cells increases when they are farther from the macro base stations. These observations highlight the particularly important role of smaller cells where macro-cell coverage is poor.





The Weighted Voronoi Diagram

So far, the discussion of Voronoi diagrams has assumed that the generator points, besides their location, have equal value, or weight. The idea of assigning distinct weight to generator points can be more useful than having uniformly weighted points in some scenarios.

Let recall the previous definition of Voronoi diagram, where a Voronoi region V_i is the intersection of the dominance regions of p_i over every other generator point in P.

The dominance region of a generator point p_i over another point p_j, where $i \neq j$ and $d_w(p_i, p_j)$ is the weighted distance between points p_i and p_j, written as

$$\mathrm{Dom}_w(p_i, p_j) = \{ p \mid d_w(p, p_i) \leq d_w(p, p_j) \}$$

The Weighted Voronoi Diagram

Let $V_w(p_i) = \bigcap_{p_j \in P \setminus p_i} \mathrm{Dom}_w(p_i, p_j)$

$V_w(p_i)$ or $V_w(i)$ is called the weighted Voronoi region, and $\mathbb{V}_w = V_w(p_1), V_w(p_2), \ldots, V_w(p_n)$ is called the weighted Voronoi diagram.

Another way to denote $\mathbb{V}_w$ is $\mathbb{V}_w(P, d_w)$ where P is the generator set with weights $W = W_1, W_2, \ldots, W_n$ and d_w is the weighted distance.

The weighted Voronoi diagram, which has its weighted distance given by

$$d_{mw}(p, p_i) = \frac{\|\mathbf{x} - \mathbf{x}_i\|}{w_i}$$

where $w_i > 0$, is called multiplicatively-weighted Voronoi diagram $\mathbb{V}_{mw}$.

The quantity d_{mw} is called the multiplicatively weighted distance.

The multiplicatively-weighted Voronoi diagram is also called Dirichlet tessellation or Apollonius model.

The Weighted Voronoi Diagram

A MW-Voronoi region is a non-empty set, it does not have to be convex or connected, and it can have a hole or holes.

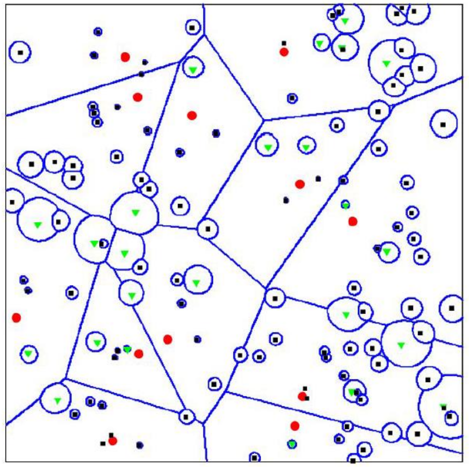
A MW-Voronoi region $V_{mw}(p_i)$ is convex if and only if the weights of adjacent MW-Voronoi regions are not smaller than the weight of p_i.

Edges in $\mathbb{V}_{mw}$ are straight lines if and only if the weights of the two interested regions are equal.

The type of Voronoi diagram, that has its weighted distance given by

$$d_{aw}(p, p_i) = \|\mathbf{x} - \mathbf{x}_i\| - w_i$$

is called additively-weighted Voronoi diagram.



Mobile Users in Multi-tier Networks

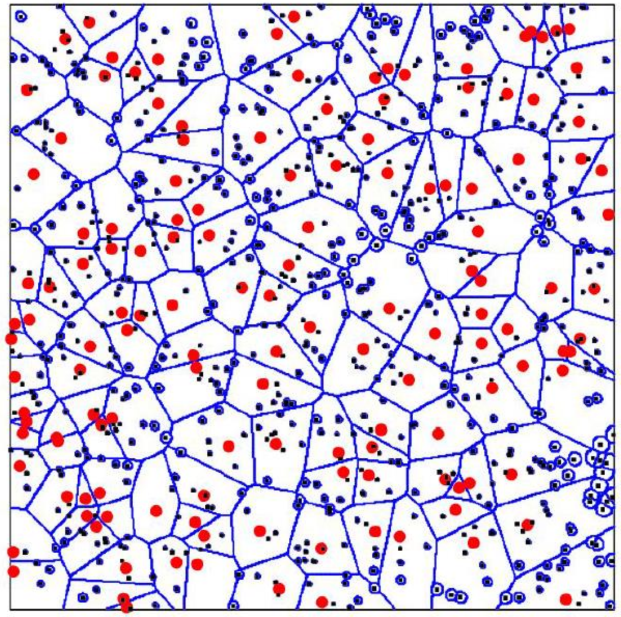
The average fraction of users served by *j*-th tier can be expressed as

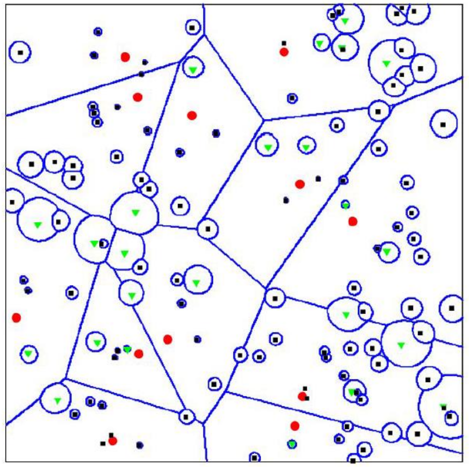
$$N_j = \frac{\lambda_j P_{tr,j}^{2/\alpha} \theta_j^{2/\alpha}}{\sum_{i=1}^{n} \lambda_i P_{tr,i}^{2/\alpha} \theta_i^{2/\alpha}}$$

$P_{tr,j}$ transmission power of the *i*-th tier,

θ SINR threshold

α path loss exponent





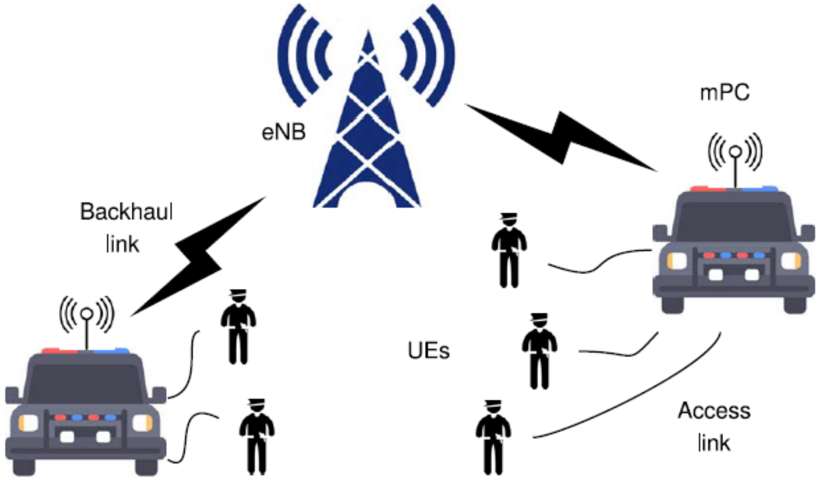
H. S. Dhillon, R. K. Ganti, F. Baccelli, and J. G. Andrews, "Modeling and analysis of k-tier downlink heterogeneous cellular networks," IEEE Journal on Selected Areas in Communications, vol. 30, no. 3, pp. 550–560, Apr. 2012.

Analytical Modelling of Disasters

5G and Public Safety Networks

Future generation networks are expected to provide infrastructure for public safety and disaster-response networks.

Heterogeneous scenario with small cells fixed or mobile.

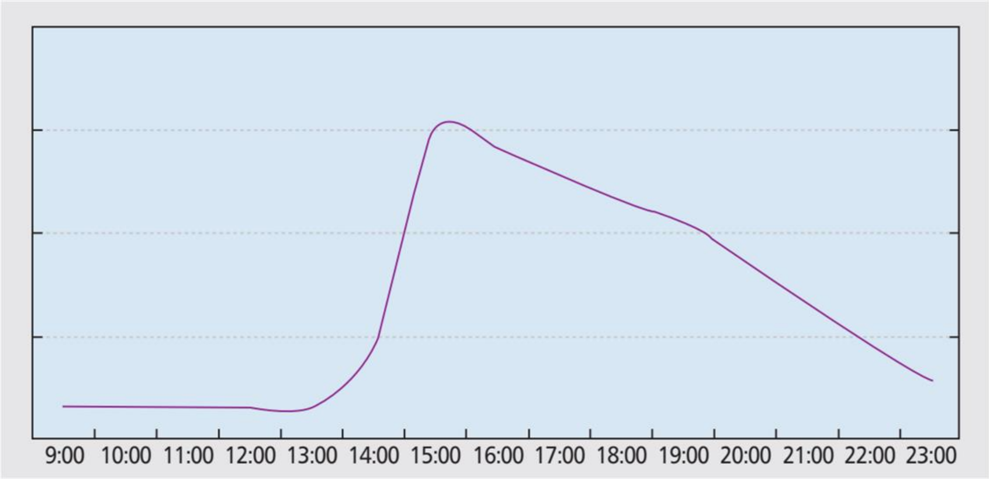


A. Jarwan, A. Sabbah, M. Ibnkahla and O. Issa, "LTE-Based Public Safety Networks: A Survey," in *IEEE Communications Surveys & Tutorials*, vol. 21, no. 2, pp. 1165-1187, Secondquarter 2019.

East Japan Great Earthquake

After East Japan Great Earthquake, one of the main problems of information network systems was traffic congestion due to the rapid traffic generation of the cellular phone system.

According to the Ministry of Internal Affairs and Communication, the numbers of call requests on cellular phones just after the earthquake were more than 10 times larger than the usual case, and the maximum call control ratio of voice communication went up to 95 percent, which means that only one person out of 20 people could use phone service. In the northern part of Japan, heavily damaged by the earthquake, the congestion in the cellular phone system was severely heavy.



Y. Shibata, N. Uchida, and N. Shiratori, “Analysis of and proposal for a disaster information network from experience of the great east japan earthquake,” IEEE Communications Magazine, vol. 52, no. 3, pp. 44–50, Mar. 2014.

East Japan Great Earthquake

The maximum congestion time was about 30 min just after the earthquake. Thus, cellular phone services were not available for a long time after the earthquake and caused serious communication problems in a wide area of Japan.

As a result, not only the damage of network devices but also the congestion of cellular phones are considered as the reasons the serious lack of disaster information, such as about rescues, evacuation shelter, and safety information occurred.

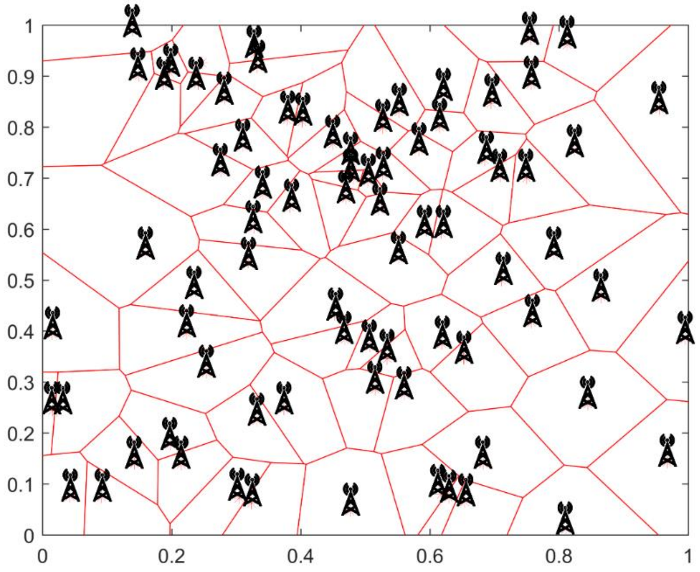
Moreover, in the disaster area such as the coastal area of Iwate prefecture, many wired networks and servers of the telecommunication companies were broken down by the huge tsunami. Therefore, fixed phone, broadband Internet services, and even the local government network system were out of service.

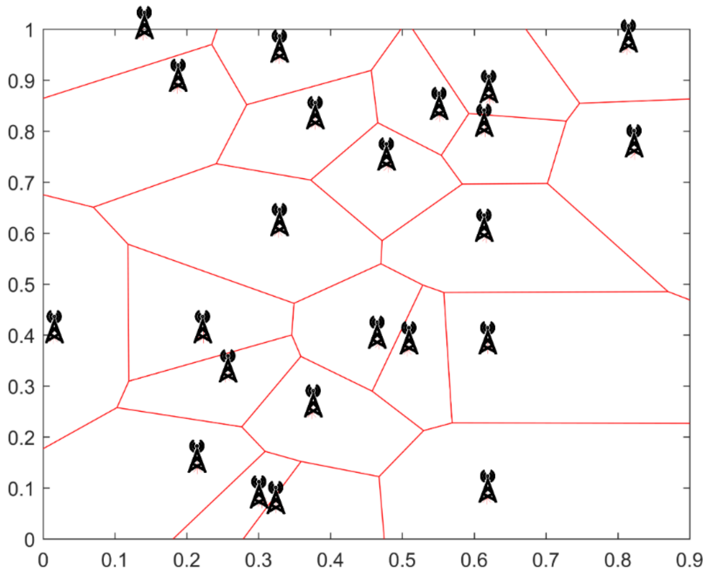
The public web services and email systems in the Iwate prefectural office as the countermeasures headquarters were also down.

Can We Analytically Model Impact of Disasters on Networks?

Disasters in Public Safety Networks

A way to model telecommunications scenarios affected by disasters or terrorist attacks it is useful to apply Independent Thinning.





A. A. Gebremariam, M. Usman, R. Bassoli and F. Granelli, “SoftPSN: Software-Defined Resource Slicing for Low-Latency Reliable Public Safety Networks,” Wireless Communications and Mobile Computing, vol. 2018, pp. 7, May 2018.

Thinning Function

Thinning one-dimensional point processes.

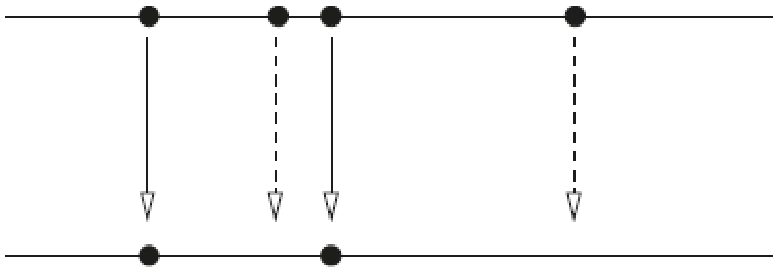
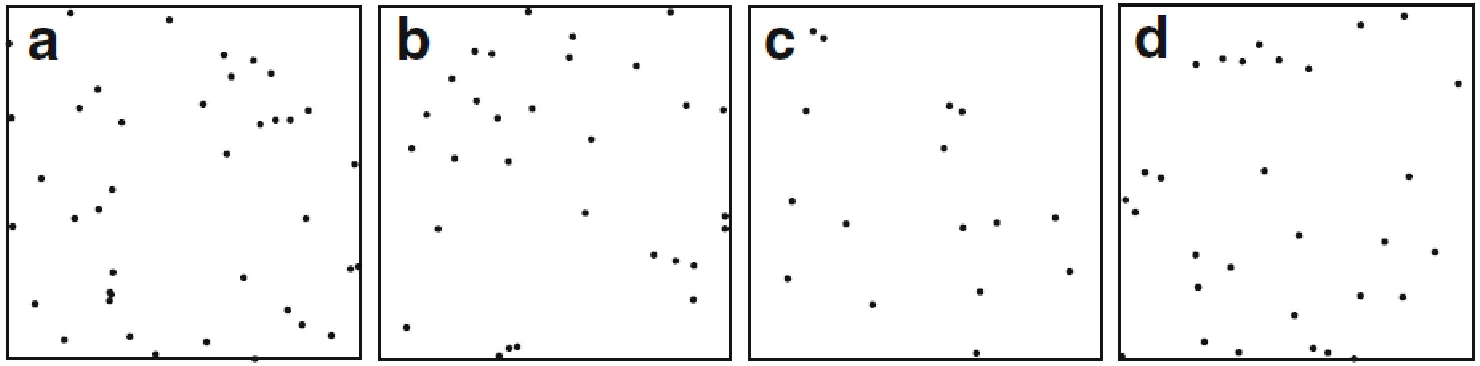


Fig. 15. Thinning a point process. Points of the original process (above) are either retained (solid lines) or deleted (dotted lines) to yield a thinned process (below).

Thinning Function

Let $g:\mathbb{R}^d \to 0,1$ be a <u>thinning function</u> and apply it to a <u>stationary PPP</u> by deleting each point $\mathbf{x}$ with probability $1-g(\mathbf{x})$, independently of all other points.

This thinning procedure generates an <u>inhomogeneous PPP</u> with intensity function $\lambda g(\mathbf{x})$.



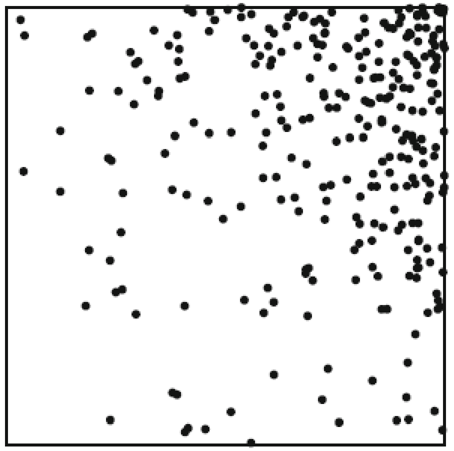


Fig. 3.45 Simulated inhomogeneous Poisson data

V. Schmidt, "Stochastic Geometry, Spatial Statistics and Random Fields - Models and Algorithms," Springer 2011.

Analysis of UAV-Enabled Disaster Recovery Networks

A large scale macro-cellular network where the locations of the base stations are modelled by a homogeneous PPP (HPPP).

Coverage holes in post-disaster result from the destruction of the cellular infrastructure. These coverage holes are modelled by location independent thinning.

Hence, the survived macro base stations will be modeled by a thinned HPPP.

The original HPPP point process, which preserves the number and the location of the holes, will then be used to model the location and number of points around which the drone-based small cellular networks are deployed.

Analysis of UAV-Enabled Disaster Recovery Networks

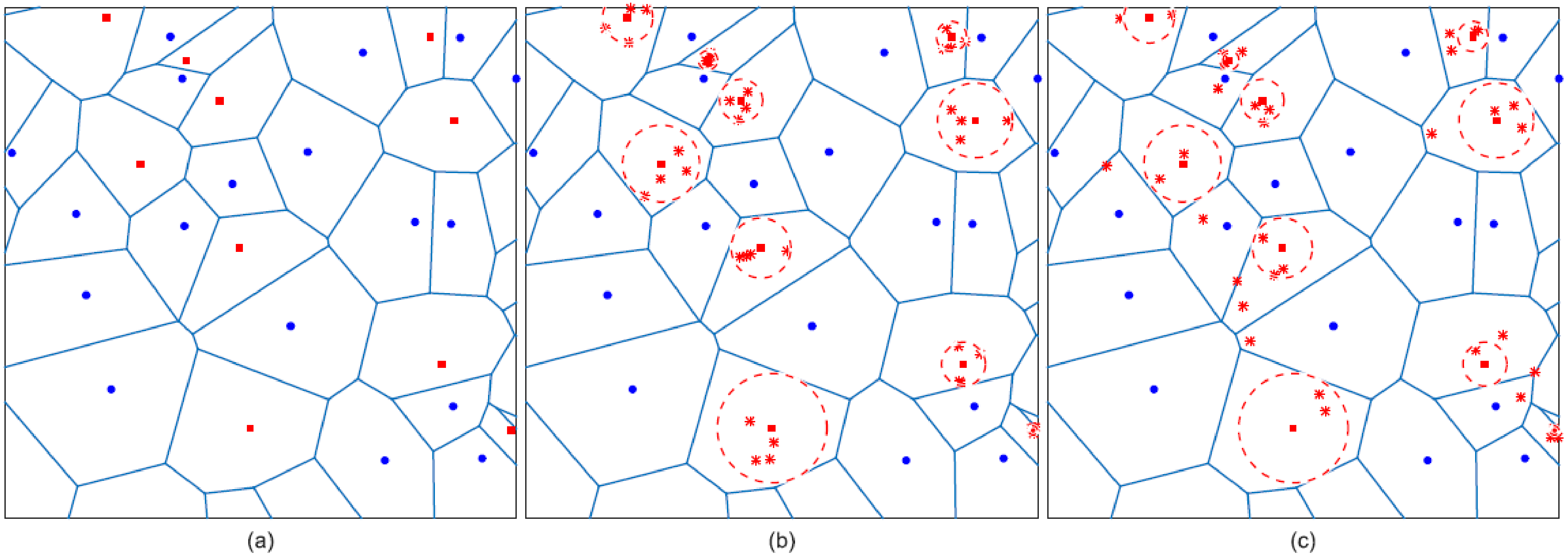


FIGURE 1. (a) Traditional cellular network where some MBSs are destroyed with probability $p^o = 0.3$. (b) Four DBSs are distributed uniformly in the two dimensional space around the center of every destroyed MBS according to a MCP model as in (5). (c) Four DBSs are distributed normally in the two dimensional space around the center of every destroyed MBS according to a TCP model as in (7). Blue circles, red squares and red stars are the retained MBSs, destroyed MBSs and the deployed DBS, respectively. A dashed circle is the radius of the deployment recovery area around the destroyed MBS.

A. M. Hayajneh, S. A. R. Zaidi, D. C. McLernon, M. Di Renzo and M. Ghogho, "Performance Analysis of UAV Enabled Disaster Recovery Networks: A Stochastic Geometric Framework Based on Cluster Processes," in *IEEE Access*, vol. 6, pp. 26215-26230, 2018.

Superposition

The superposition of two independent Poisson point processes Φ_1 and Φ_2 , say uniform Poisson processes of intensity λ_1 and λ_2 respectively, is a uniform Poisson process of intensity $\lambda_1 + \lambda_2$.

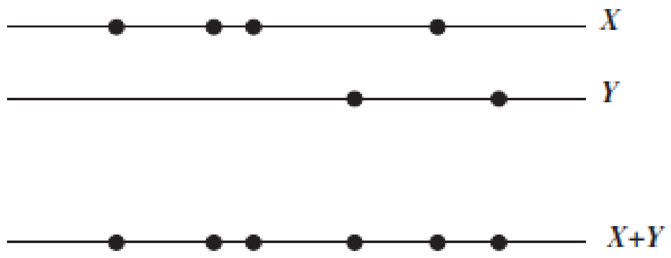
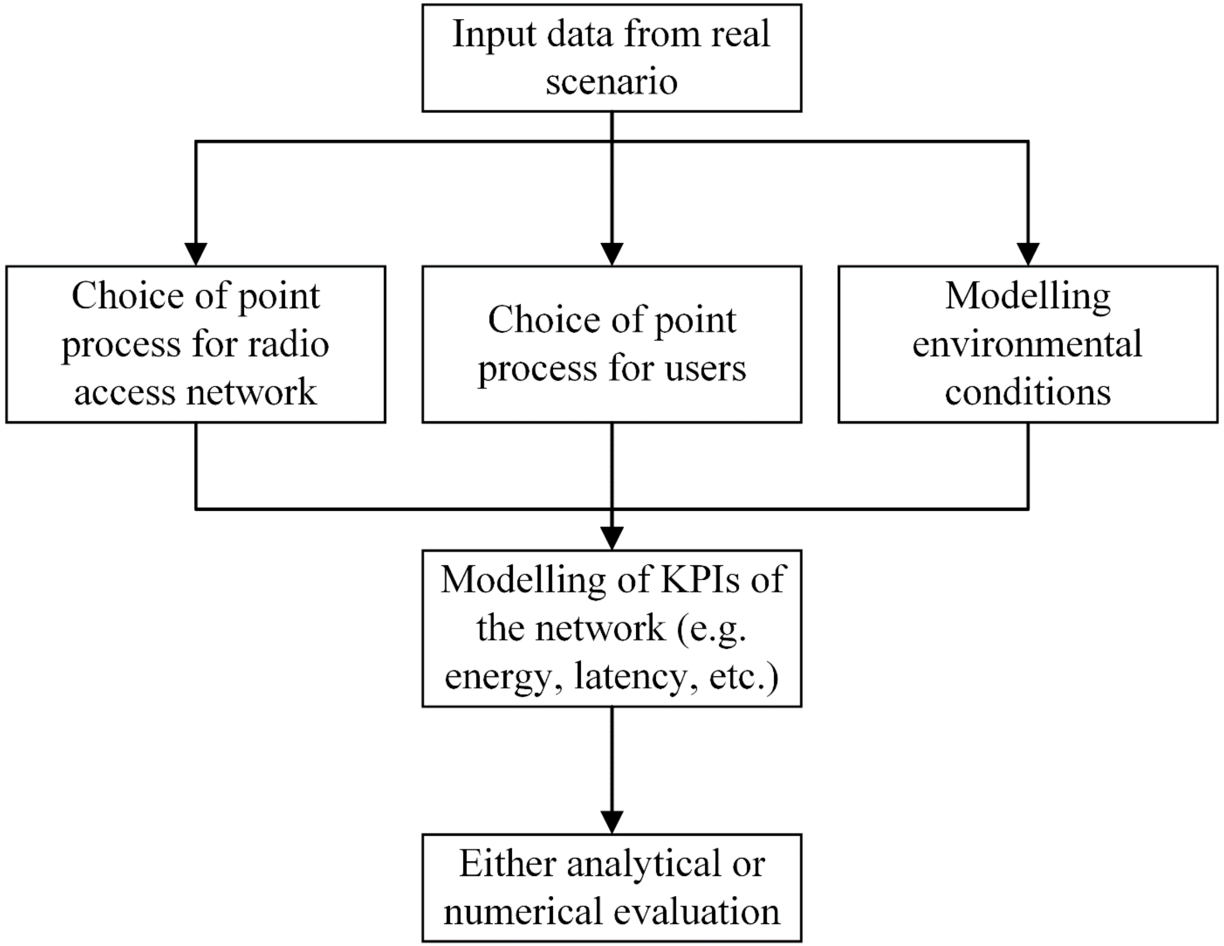


Fig. 17. Superposition of two point processes

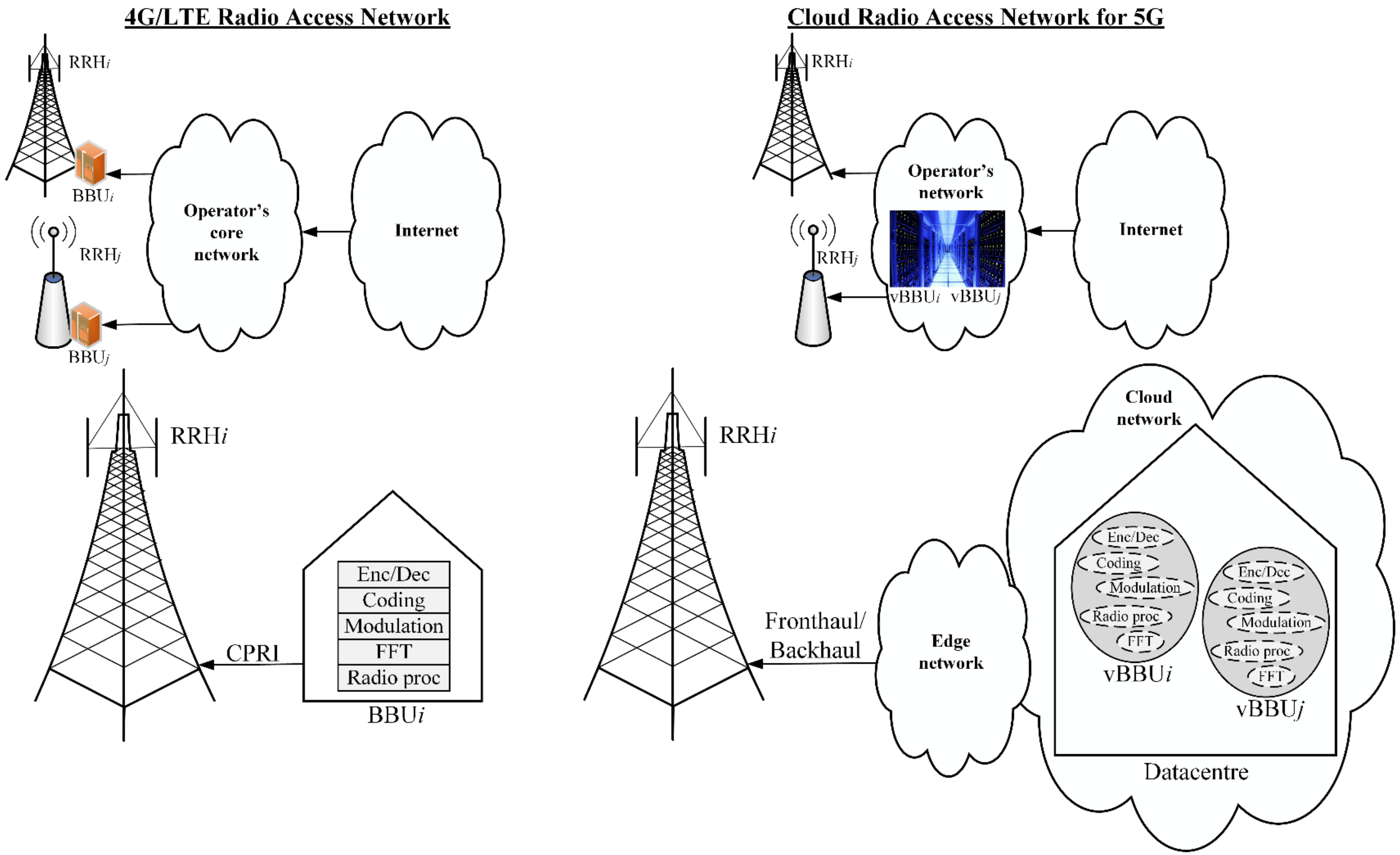
M. Franca, “Stochastic Geometry,” Springer, 2004.

Possible Logic Modelling Procedure



An Analytical Model for Future Generation Networks

An Example: The Concept of Cloud Radio Access Network



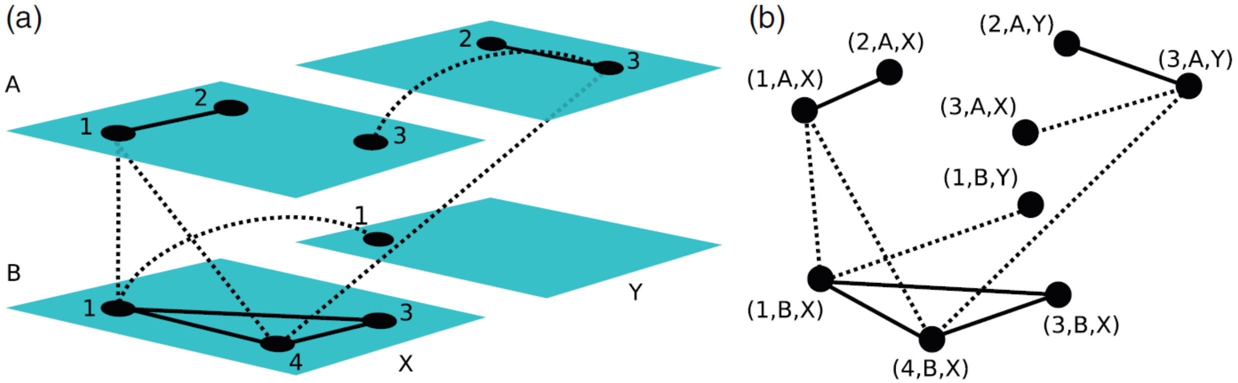
R. Bassoli, F. Granelli, S. T. Arzo and M. Di Renzo, "Towards 5G Cloud Radio Access Network: An Energy and Latency Perspective," in Transactions on Emerging Telecommunications Technologies, Jun. 2019.

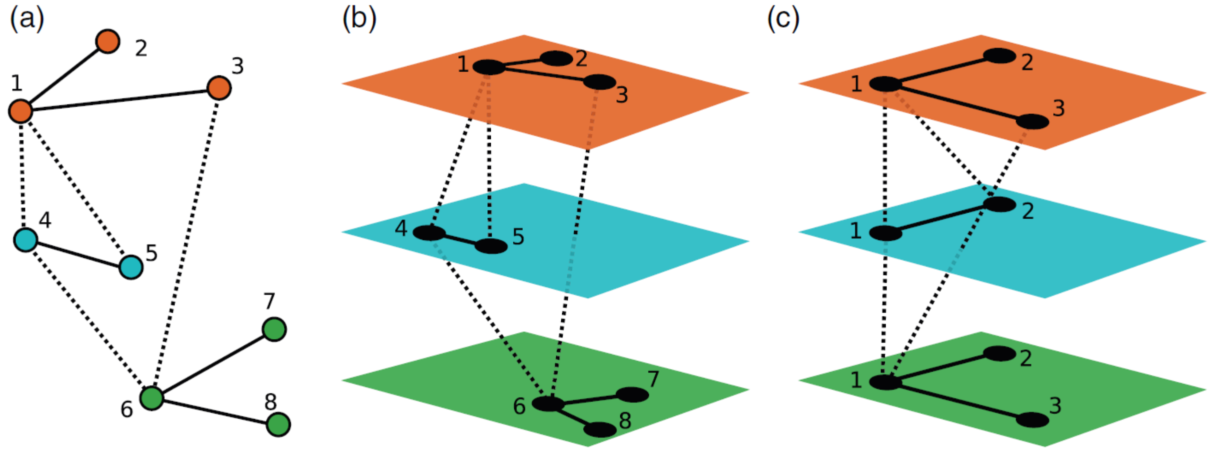
Modelling Future Generation Networks

Four main open issues in theoretical research about 5G C-RAN:

1. 5G C-RAN is not modelled as heterogeneous system with spatial information. The study of C-RAN, in the context of 5G, cannot neglect the characterisation of SINR which requires knowledge of network geometry.
2. Virtualisation of RAN is not contextualised in a framework, that models the actual architecture of a data centre.
3. The evaluation of power and latency does not consider all the parts of the network. Especially, the different impact of edge and cloud computing or the specific architecture of the data centre are frequently neglected.
4. In 5G C-RAN investigation, it is not analysed the trade-off between power consumption/energy efficiency and latency.

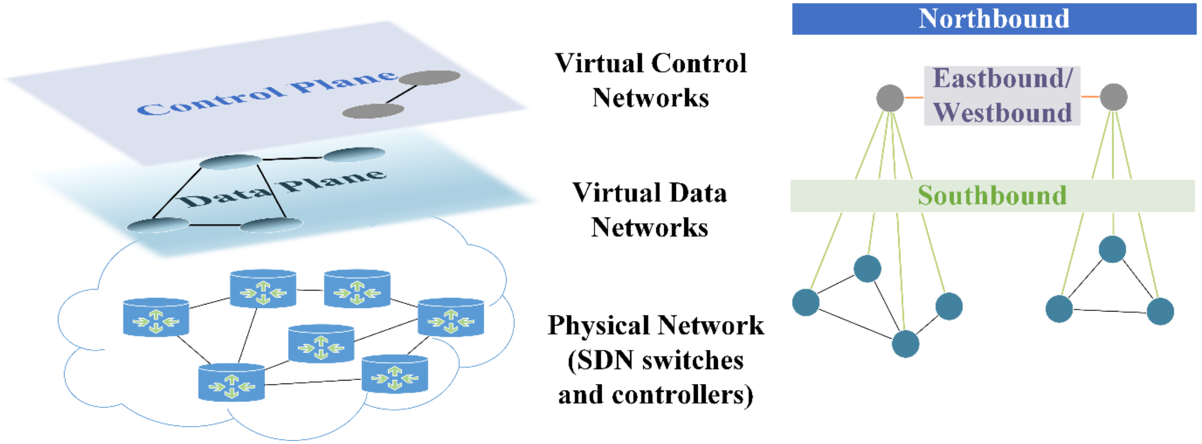
Hypergraphs and Wireless Networks

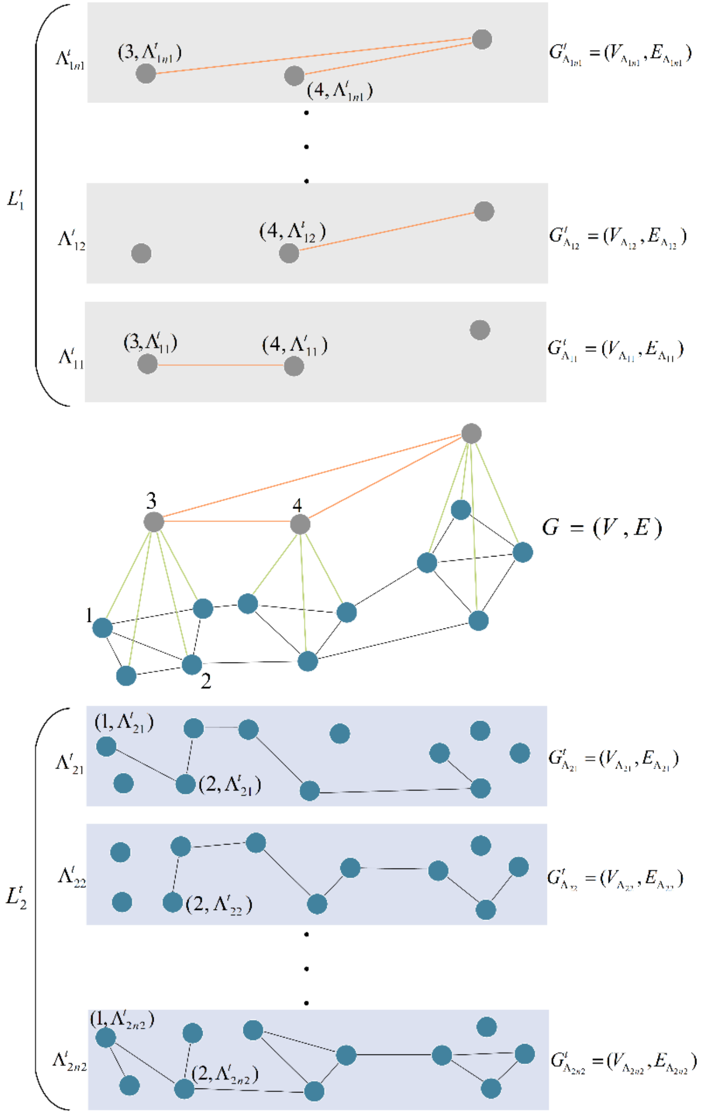




Mikko Kivelä, Alex Arenas, Marc Barthelemy, James P. Gleeson, Yamir Moreno, Mason A. Porter, Multilayer networks, *Journal of Complex Networks*, Volume 2, Issue 3, September 2014, Pages 203–271, https://doi.org/10.1093/comnet/cnu016

Multilayer Graphs and Future Generation Networks

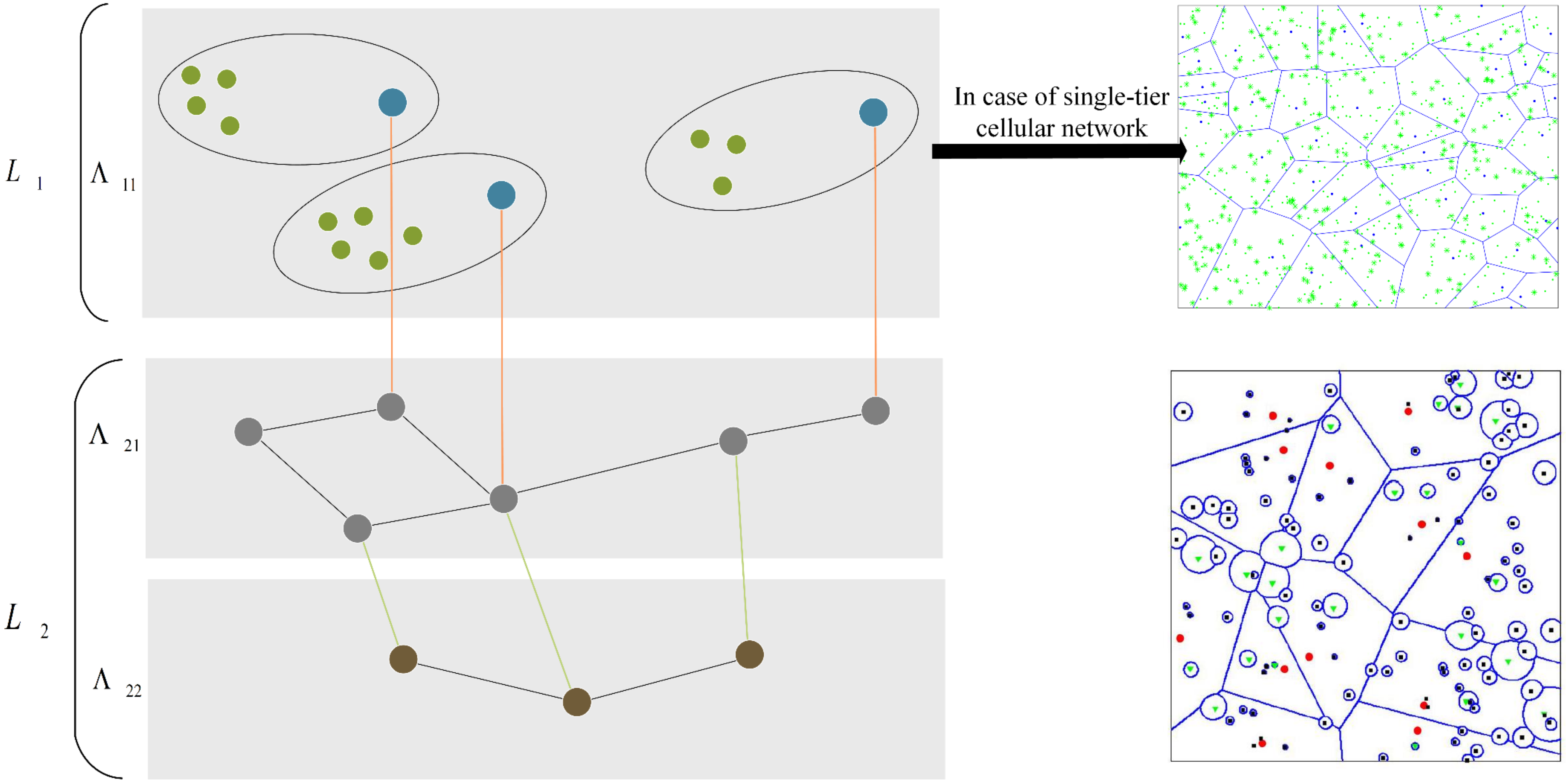




R. Bassoli and F. Granelli, "An Algebraic Approach to Network Slicing," *European Wireless 2019; 25th European Wireless Conference*, Aarhus, Denmark, 2019, pp. 1-6.

What Model for Future Generation Networks?

Random Multilayer Hypergraph



R. Bassoli, F. Granelli, S. T. Arzo and M. Di Renzo, "Towards 5G Cloud Radio Access Network: An Energy and Latency Perspective," in Transactions on Emerging Telecommunications Technologies, Jun. 2019.

Random Multilayer Hypergraph for Future Generation Networks

Let $H = (X, E)$ be a planar hypergraph representing the physical network,

X is the set of nodes

E is the set of non-empty subsets of X, called hyperedges.

Next, the set X can be partitioned into subsets $X = (X_1, X_2, \ldots)$ respectively referred to mobile end users, base stations, network nodes and internal nodes of data centres' network (hosting the virtual baseband units).

Random Multilayer Hypergraph for Future Generation Networks

Let $\mathcal{H} = (X, E, X_{\mathcal{H}}, E_{\mathcal{H}}, L)$ be a multilayer random hypergraph where

X is the set of random nodes, which can be distributed according to either random point processes (e.g. base stations) or deterministic spatial distributions (e.g. wired operator's network);

E is the set of random hyperedges, whose cardinality can be defined by either Voronoi tessellation in $\mathbb{R}^2$ (e.g. wireless cellular networks) or deterministic values (e.g. links in wired networks);

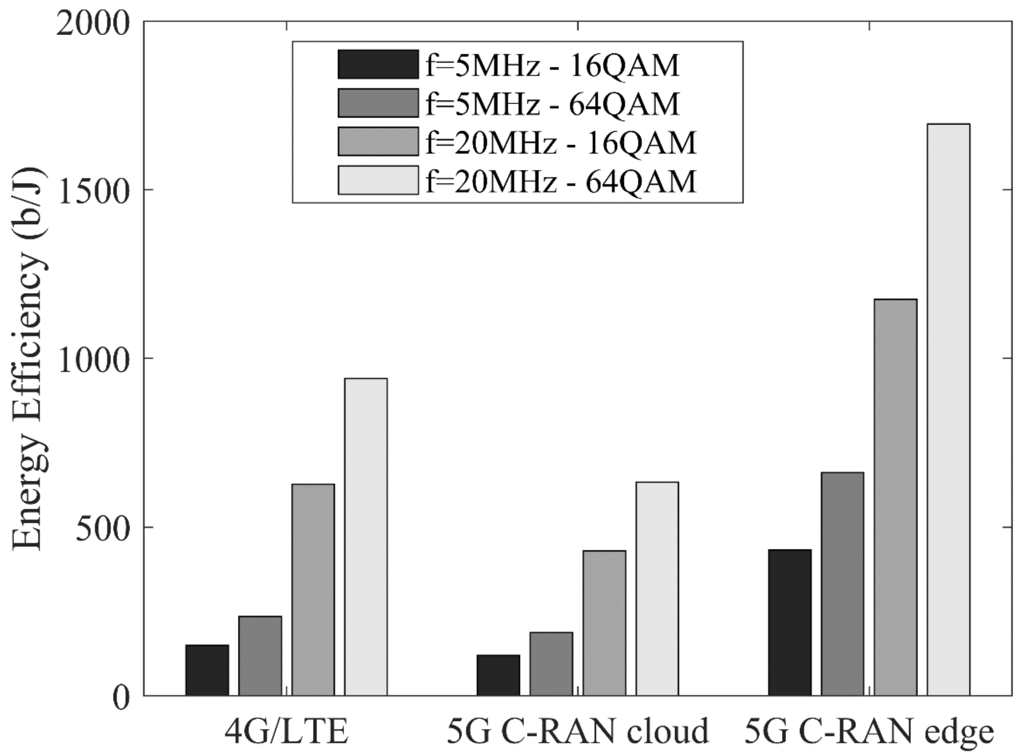
$L = \{L_1, \dots, L_a\}$ is the set of layers, where *a* is the number of aspects; each layer can be a set of sub-layers Λ_{ij} , where *i* is the number of layer it belongs to and *j* is the number of sub-layer $j = 1, \dots, |L_i|$

$X_{\mathcal{H}}$ is the set of node-layer elements;

$E_{\mathcal{H}}$ is the set of hyperedge-layer elements

An Example: Energy Efficiency and Latency in Future Generation Networks

$$P_{tot5G} = P_{RAN} + P_{bh} + P_{net} + P_{dc}$$



R. Bassoli, F. Granelli, S. T. Arzo and M. Di Renzo, "Towards 5G Cloud Radio Access Network: An Energy and Latency Perspective," in Transactions on Emerging Telecommunications Technologies, Jun. 2019.

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2. H. ElSawy, E. Hossain and M. Haenggi, "Stochastic Geometry for Modeling, Analysis, and Design of Multi-Tier and Cognitive Cellular Wireless Networks: A Survey," in *IEEE Communications Surveys & Tutorials*, vol. 15, no. 3, pp. 996-1019, Third Quarter 2013.
3. H. ElSawy, A. Sultan-Salem, M. Alouini and M. Z. Win, "Modeling and Analysis of Cellular Networks Using Stochastic Geometry: A Tutorial," in *IEEE Communications Surveys & Tutorials*, vol. 19, no. 1, pp. 167-203, Firstquarter 2017.
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5. S. Zhang, J. Liu and W. Sun, "Stochastic Geometric Analysis of Multiple Unmanned Aerial Vehicle-Assisted Communications Over Internet of Things," in *IEEE Internet of Things Journal*, vol. 6, no. 3, pp. 5446-5460, June 2019.

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6. R. Bassoli and F. Granelli, "An Algebraic Approach to Network Slicing," *European Wireless 2019; 25th European Wireless Conference*, Aarhus, Denmark, 2019, pp. 1-6.
7. R. Bassoli, F. Granelli, S. T. Arzo and M. Di Renzo, "Towards 5G Cloud Radio Access Network: An Energy and Latency Perspective," in Transactions on Emerging Telecommunications Technologies, Jun. 2019.