An Algebraic Approach to Network Slicing

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***Abstract*—Virtualisation and virtual network slicing represent the main paradigms to enable efficient and effective end-to-end service provisioning in future fifth generation and beyond networks. While practical aspects and implementation have been extensively investigated, the development of theoretic models to enable the design and analysis of advanced slicing algorithms has only recently started. However, even if existing models are useful to analyse specific aspects of network slicing performance in specific topologies, they still outline limitations and drawbacks for providing an actual theoretical basis for network slicing. This article proposes a novel general model for network slicing based on multilayer graphs, linear algebra and algebraic graph theory. The proposed framework generalises specific legacy models by allowing a more comprehensive study of different perspectives of network slicing in future generation networks.**

***Index Terms*—5G, virtualisation, network slicing, software-defined networking, multilayer graphs, algebraic geometry.**

I. Introduction

The vision of future fifth generation (5G) and beyond (B5G) networks depicts a highly heterogeneous ecosystem of technologies, networks, protocols and services, which will have to coexist, to work and to adapt in a effective and efficient manner. In such context, network virtualisation represents the main enabler to guarantee successful network management and flexibility. The pivotal networking paradigm to support the actual realisation of virtualisation and 'programmable networks' is software-defined networking (SDN) [1]. Software-defined networking represents the baseline for network slicing [2]–[4]. This concept mainly consists in end-to-end virtual network provisioning to efficiently satisfy the diversified and dynamic service requirements of 5G and B5G networks.

The idea of virtual network slicing was proposed in 2013. Initially, research was mainly focused on practical characteristics, architectures and real implementation. Nevertheless, investigation was recently concentrated on theoretical and algorithmic aspects [5]. Such change of perspective was necessary to better understand how to allocate resources and slices according to network and service changes. However, existing theoretical models still present some drawbacks and limitations in the analysis of complex environments like virtual networks.

A physical network is usually represented as an undirected weighted graph, whose weights model the different network resources taken into account in the specific study. Then, the works in the literature differentiate according to how the problems of virtualisation and slicing of resources are interpreted and modelled.

A first group simply considers network slices as sets of parameters [6]. While this approach can be useful in some specific scenarios (e.g. networks composed by mobile users and base stations [6]), it does not consider the topological information and the time-dependent spatial characteristics of the network. Furthermore, it is hard to describe the characteristics of online mapping and assignment of resources with this model.

Next, a second family uses overlay networks to distinguish between nodes and links in physical and virtual networks. This concept has been clearly explained in [7]. The idea is to generate virtual network topologies on the physical network one. An application of this concept can be found in [8]. The drawbacks of this perspective is the lack of flexibility to provide insights on the characteristics and behaviour of online slicing in very complex topologies.

Another approach [9] modelled virtual network slicing as an embedding problem. In particular, virtual networks are undirected graphs embedded in the one of the physical network. Network slicing can be also studied as a set of flows and nodes assignment problems [10]. Nevertheless, these theoretical descriptions hardly represent the complex characteristics of the time-dependent structure of virtual networks. Next, [11] considered network slices as collections of virtual nodes and links supporting customised services, which are mapped on the physical network (a weighted undirected graph). This approach may not be effective to understand and to investigate the complex structure and behaviour of the virtual network topology.

Recently, a useful model [5] has been designed to describe and to study network slicing: it went towards a theoretical modelling to analyse efficiently both virtual networks' behaviour and algorithmic aspects of slicing. This novel model is mainly based on virtual network embedding (VNE) [12], which enables the optimisation of the process of mapping virtual resources onto physical ones. This approach also includes slicing as a constrained optimisation problem. Then, [13] enhanced the VNE-based model by proposing a multi-layer VNE model, in which each layer is an undirected graph (a virtual network), which is mapped onto the undirected graph of the physical network. While this approach is mathematically rigorous and detailed, it presents some drawbacks such as the fact that the VNE problem is NP-hard [12], so it only requires heuristic or meta-heuristic solutions.

Overall, all the previous theoretical studies on network slicing do not include the heterogeneous characteristics of end-to-end services of future 5G and B5G networks (not only in terms of capacity but also in terms of latency and reliability). In addition, all the previous models do not consider the differentiation between control and data plane, the interconnections between virtual control and data networks, and the possibility of slicing the control plane (in case of networks with distributed SDN controllers).

As a consequence, and to the best of authors' knowledge, the novel achievements of this article are:

- the rigorous mathematical definition of a model to investigate network slicing of backbone networks in the context of 5G and B5G. This novel approach is based on multilayer networks [14], which are very flexible mathematical objects to model the time-dependent characteristics of SDN-based virtual networks and online network slicing.
- the proposed model combines multilayer networks, linear algebra and algebraic graph theory. The application of algebraic graph theory in the context of multilayer graphs opens the possibility of applying tools from algebraic geometry on network slicing. It is the first time virtual network structure can be modelled as an affine variety composed by the polynomials of the virtual networks, which allowed the discovery of a correlation between the greatest common divisor (gcd) of the affine variety of a virtual network and the number of virtual network slices at each time interval.
- the theoretical study of virtual networks' properties and network slicing performance considering 5G and B5G heterogeneous end-to-end services, with different demands in terms of capacity, latency and reliability.

This article has the following structure. Section II expounds the proposed model in detail. Next, Section III evaluates numerically some important aspects of slicing in 5G and B5G, while studying algebraic properties of virtual networks.

II. Virtual Networks as Multilayer Graphs

Let $G = (V, E)$ be the planar graph, reproducing the physical infrastructure of 5G and B5G backbone network. Let V be the set of vertices and let E be the set of edges. In particular, the former is partitioned into three main subsets V_1, V_2 and V_3 which are respectively the set of global SDN controllers, local SDN controllers and SDN switches.

Let $M = (V, E, V_M^t, E_M^t, L^t)$ be a multilayer loopless graph where

- V is the set of nodes ('physical' SDN switches and controllers);
- E is the set of edges ('physical' links among all SDN switches and controllers);
- $L^t = \{L_1^t, \ldots, L_a^t\}$ is the set of layers according to the number a of *aspects*; then, subsets $L_i^t = \{\Lambda_{i1}^t, \Lambda_{i2}^t, \ldots, \Lambda_{in_i}^t\}$ are the sets of elementary layers Λ_{ij}^t, given the ith aspect; the variables $n_1, \ldots, n_i, \ldots, n_a$ represent the number of elementary layers per each aspect. The timestamp $t \in \mathcal{T}$ conveys the time-dependence characteristic of layers, where $\mathcal{T}$ is the set of all time stamp values;

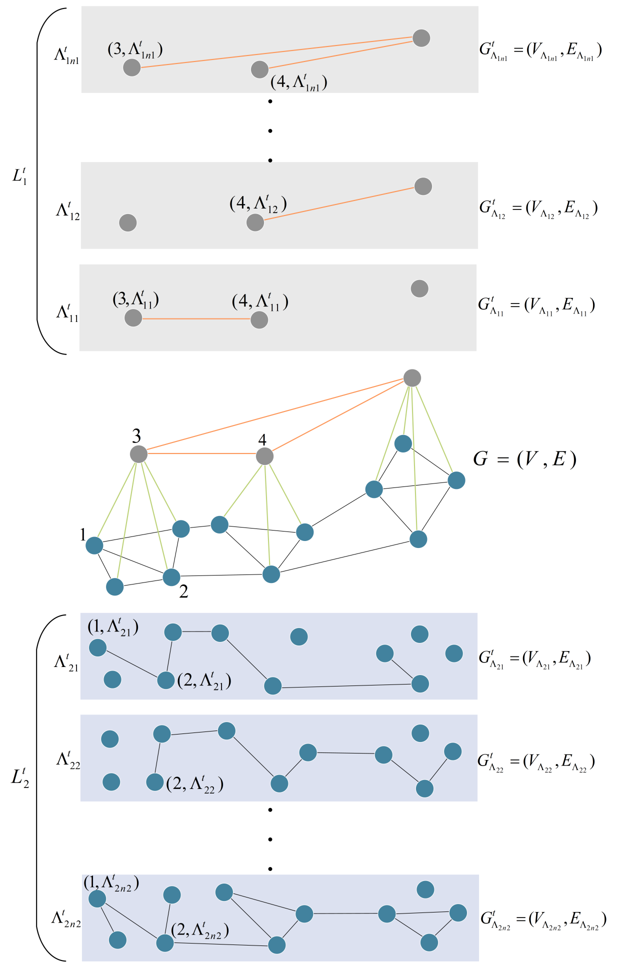

Fig. 1. Graphic representation of the multilayer graph model of a virtual network, performing network slicing. Green links between elementary control layers and elementary data layers are omitted because of clarity in the graphical representation. Only some labels are represented in the graph just as examples.

- V_M^t is the set of node-layer elements, in which each node in V can differently appear since referred to the respective elementary layer (considering time-dependence);
- E_M^t is the set of edge-layer elements, in which each edge in E can differently appear because it is referred to the respective elementary layer (considering time-dependence).

The timestamp t and the assumptions on L^t, V_M^t and E_M^t reflect the intrinsic nature of virtualisation and slicing. The total number of layers of M at a given time is obtained as $n_{tot} = \sum_{i=1}^{a} n_i$. Next, each elementary layer represents a planar graph $G^t = (V_{\Lambda_{ij}}, E_{\Lambda_{ij}})$, where $V_{\Lambda_{ij}} \subseteq V$ and $E_{\Lambda_{ij}} \subseteq E$ (because of clarity, the notation implies timestamp t for $V_{\Lambda_{ij}}$, $E_{\Lambda_{ij}}$ and when Λ_{ij}^t is used as an index). Planar graphs can have vertices with zero degree (if the nodes do not belong to the slice).

Figure 1 depicts an example of this multilayer graph model to clarify its visual structure and notation.

Let $\mathbf{A}_G = (a_{ij})$ be the adjacency matrix of size $|V| \times |V|$

obtained from the ordered vertex set $(<, V)$ of planar loopless graph $G = (V, E)$, where $<$ is a binary relation over V. Next, let $\mathbf{X}_G = (x_{ij})$ be the incidence matrix of size $|V| \times |E|$, referred to the planar loopless graph $G = (V, E)$. The definition of adjacency and incidence matrices of M require the generalisation to tensor theory [15]. Hence, we define the fourth order adjacency tensor and incidence tensor, which can be seen as the four-dimensional arrays $\mathbf{A}^t_M = (a^t_{ijk})$ and $\mathbf{X}^t_M = (x^t_{ijk})$ (the third dimension is referred to the number of elementary layer and the time stamp t identifies the fourth dimension of time). The former (tensor $\mathbf{A}^t_M$) has size $|V| \times |V| \times |L| \times |L_1| \times \ldots \times |L_a|$, while the latter (tensor $\mathbf{X}^t_M$) has size $|V| \times |E| \times |L| \times |L_1| \times \ldots \times |L_a|$. Since the number of layers (slices) changes with time, $\mathbf{A}^t_M$ and $\mathbf{X}^t_M$ have variable size. Next, let's define *edge attributes*:

- the function $c \colon E \to \mathbb{R}$, which associates a weight to the edges of G (the capacity of the 'physical' links measured in b/s);
- the function $\tau \colon E \to \mathbb{R}$, which associates a weight to the edges of G (the delay of the edges measured in s);
- the function $\rho \colon E \to \mathbb{R}$, which associates a weight to the edges of G (the reliability of the edges defined as $\rho = 1 - P_e$, with P_e the failure probability of an edge).

The two functions c and τ allow the definition of two weight matrices referred to G, the *capacity matrix* $\mathbf{C}_G = (c_{ij})$ and the *delay matrix* $\mathbf{T}_G = (\tau_{ij})$. These matrices have the same size of the adjacency matrix $\mathbf{A}_G$. Especially, the delay matrix has members calculated as the sum of two components $\mathbf{T}_G = \mathbf{T}_p + \mathbf{T}^t_{tr}$:

- $\mathbf{T}_p = (\tau_{p\,ij})$ is a matrix of constants, which identifies the propagation delay (dependent on the distance weight between nodes);
- $\mathbf{T}^t_{tr} = (\tau^t_{tr\,ij})$ is the matrix of transmission delay, inversely proportional to the available link capacity at time t;

This proposed latency description of matrix $\mathbf{T}_G$ can be further enhanced without loss of generality of the overall model.

Next, the function ρ permits the definition of a *reliability matrix* associated to G, which is defined as $\mathbf{F}_G = (\rho_{ij})$. Thus, the overall reliability of a path is the product of the reliabilities of the single edges in the path since the probability of each link is assumed independent of the other ones.

An *end-to-end service* is identified by the ith commodity flow, thus a quadruple $(s_i, \sigma_i, \alpha_i, D_i)$, where $s_i \in S$ is the source (S is the set of sources), $\sigma_i \in \Sigma$ is the sink (Σ is the set of sinks) and α_i is the indicator function $1_{\mathcal{K}} \colon K \to \{0, 1\}$, which assumes value 1 if the commodity k_i belongs to subset $\mathcal{K} \in K$ of commodities with elastic demand set (value 0 means inelastic demand). Then, let D_i be the *demand set*, which defines the requirements in terms of capacity, latency and reliability and priority for that specific application: if commodity is elastic $D_i = \{[\tilde{c}_{min}, \tilde{c}_{max}], [\tilde{\tau}_{min}, \tilde{\tau}_{max}], [\tilde{\rho}_{min}, \tilde{\rho}_{max}], \beta\}$ (the first three requirements are ranges), otherwise (inelastic case) the set becomes $D_i = \{\tilde{c}, \tilde{\tau}, \tilde{\rho}, \beta\}$, with constant first three members. The priority β is a value in the range $[0, 1]$, which is used to classify the serving priority of the commodity; the sum of all the priorities is 1. The coexistence of multiple services is called a multicommodity flow and $K = \{k_i\}$ (with $i = 1, \ldots, m$) is the set of m commodities.

Let $\mathbf{R}^t_M = (r^t_{ijk})$ be the 4-dimensional *routing tensor* at time t, which defines the paths of the flows for each layer (this routing tensor refers to slices), which determines the set of links crossed by a flow and eventually the percentage of flow on different links in parallel (in case of load balancing).

Next, let $\mathbf{Z}^t_M = (z^t_{ijk})$ be the Ingress-Egress (IE) *traffic tensor* (volume of traffic per virtual edge in a slice in terms of bits, for the time interval $[t, t + \Delta t] \subset \mathcal{T}$). The diagonal members of the tensor are assumed to be zeros since graph is loopless.

Axiom 1. Traffic tensor $\mathbf{Z}^t_M$ is stationary in the interval of measurement and the measure is error-free. The routing matrix remains constant in the interval of measurement.

Axiom 2. Let $\mathcal{T}$ be the set of timestamps and let $\bar{t}$ be a selected member in the set $\mathcal{T}$. Then, $\forall \bar{t} \in \mathcal{T}$ the condition $c_{ij} \geq \sum_{k=1}^{n_{tot}} z^{\bar{t}}_{ijk}$.

Axiom 3 (Existence of a Slice)**.** Given the graph G of the physical network, let $S_i \subset S$ be a subset of sources and let $\Sigma_i \subset \Sigma$ be a subset of destinations such that $S_i, \Sigma_i \subset V$. Next, let $H_{S_i,\Sigma_i} = \{H_1, \ldots, H_h\}$ be the set of all possible h paths between set of sources S_i and set of sinks Σ_i such that each member H_i connects elements of S_i and Σ_i, with $H_i \subseteq E$. Let E_{H_i} be the set of edges in the path H_i. Then, the $\Lambda_{i,j}$th slice (or the graph $G^t_{\Lambda_{i,j}} = (V_{\Lambda_{i,j}}, E_{\Lambda_{i,j}})$) exists and includes H_i if and only if

$$\begin{cases} \forall e_{H_i} \in E_{H_i}, c_{e_{H_i}} \geq \sum_{k=1}^{n_{tot}} z^{\bar{t}}_{e_{H_i}} + \tilde{c} \\ \tilde{\tau} \geq \sum_{i=1}^{|E_{H_i}|} \tau_i \\ \tilde{\rho} \geq \prod_{i=1}^{|E_{H_i}|} \rho_i \end{cases} \tag{1}$$

Axiom 3 is used as requirement to create a slice and also the condition of admission for an end-to-end service to that existent network slice.

A. Algebraic Formulation

According to algebraic graph theory [16], each graph can be characterised by a polynomial obtained from its adjacency matrix: this *characteristic polynomial* is in the form $f(x) = \det(x\mathbf{I} - \mathbf{A}_G)$, where $\mathbf{I}$ is the identity matrix. Next, the *eigenvalues of a graph* are the roots of its characteristic polynomial such that $f(x) = 0$.

Thus, the multilayer graph M, which consists of all the planar graphs $G^t = (V_{\Lambda_{ij}}, E_{\Lambda_{ij}})$, representing virtual slices, can be seen as a time-varying set of polynomials in one variable $f^t_{11}(x), \ldots, f^t_{1n_i}(x), \ldots, f^t_{ij}(x), \ldots, f^t_{an_a}(\mathrm{x})$. This assumption relates network slicing and slices' time-dependent structure to set of polynomials and, more generally, to algebraic geometry and varieties.

Let M be the multilayer loopless graph, defined in the previous section. Each elementary layer Λ^t_{ij} has a characteristic polynomial $f^t_{ij}(x) = \sum_m a_m x^m$, with coefficients a_m in $\mathbb{R}$.

Then, all the elementary layers in L^t define a set of polynomials, which lie in $\mathbb{R}[x]$. Finally, it is possible to define algebraic-geometric statements for virtual network slicing [17].

Definition 1. Let $\mathbb{R}$ be the field of coefficients, and let $f_{11}^t, \ldots, f_{1n_i}^t, \ldots, f_{ij}^t, \ldots, f_{an_a}^t$ be the polynomials of the virtual network, which lie in $\mathbb{R}[x]$. Then, $\mathcal{V}(f_{11}^t, \ldots, f_{an_a}^t) = \{(a_1, \ldots, a_n) \in \mathbb{R}^n : f_{ij}^t(a_1, \ldots, a_n) = 0\}$. So, $\mathcal{V}(f_{11}^t, \ldots, f_{an_a}^t)$ is the *affine variety of the virtual network* defined by $f_{11}^t, \ldots, f_{an_a}^t$.

Definition 2. Let $\mathcal{V}(f_{11}^t, \ldots, f_{an_a}^t)$ be the time-varying affine variety of the virtual network. The virtual network is *consistent* if $\mathcal{V}(f_{11}^t, \ldots, f_{an_a}^t) \neq \emptyset$, i.e. all equations $f_{11}^t = \ldots = f_{1n_i}^t = \ldots = f_{ij}^t = \ldots = f_{an_a}^t = 0$ have a common solution. Moreover, this affine variety is *finite* and has an explicit solution.

Axiom 4. Let $f_{11}^t, \ldots, f_{1n_i}^t, \ldots, f_{ij}^t, \ldots, f_{an_a}^t$ be the set of characteristic polynomials of the virtual network, which lie in $\mathbb{R}[x]$. Let f_G be the characteristic polynomial of the physical network. All the polynomials f_{ij}^t have same degree $\deg(f_{ij}^t) = \deg(f_G)$.

Definition 3 (Greatest Common Divisor)**.** A greatest common divisor of a virtual network M at time t, is the greatest common divisor of polynomials $f_{11}^t, \ldots, f_{1n_i}^t, \ldots, f_{ij}^t, \ldots, f_{an_a}^t \in \mathbb{R}$, which is a polynomial $g^t(x)$ such that

- g divides $f_{11}^t, \ldots, f_{1n_i}^t, \ldots, f_{ij}^t, \ldots, f_{an_a}^t$.
- If $p(x)$ is another polynomial which divides $f_{11}^t, \ldots, f_{1n_i}^t, \ldots, f_{ij}^t, \ldots, f_{an_a}^t$, then p divides g.

When g has these properties, it is possible to define the *greatest common divisor of a virtual network* as $g^t(x) = \gcd(f_{11}^t, \ldots, f_{1n_i}^t, \ldots, f_{ij}^t, \ldots, f_{an_a}^t)$.

III. Numerical Results

This section presents the setup characteristics and the results of MATLAB simulations of the model described above. In particular, the proposed theoretical model is tested on two specific topologies.

A. Preliminary discussion on the setup

Existing 4G networks are designed to support mostly mobile broadband services. The requirements of such services are not very stringent and the main focus in service level agreements (SLAs) is referred to the offered / average peak data rate. However, 5G is expected to support an ecosystem of technologies and services, whose requirements will be extremely diverse. Currently, no data is available about real implementation of 5G-type traffic,composed by Extreme Mobile Broadband (xMBB), ultra-reliable Machine-Type Communications (uMTCs) and massive Machine-Type Communications (mMTCs): thus no realistic traffic models are available to describe the process of arrivals and departures of end-to-end services. Because of this assumption and to provide a simple proof-of-concept, the following numerical evaluations just consider uniformly random arrivals/departures at each simulation time interval. Other models can be easily used.

The slice broker [18] is the entity which performs slicing and admission control. When a service request arrives, the slice broker checks what paths satisfy the requirements of the demand set D_i. Given a source and a destination, it uses the graph's adjacency list to evaluate Axiom 3. If a slice exists, the service is admitted. There are two cases:

1) if a slice with the same topology already exists, the service joins that slice;
2) if the required slice's topology does not exist, the slice broker creates this new slice.

If Axiom 3 is not satisfied, the admission request is denied, and the slice broker serves the next one according to the priority β. The service priority follows the reasonable order: uMTC, mMTC and xMBB. This slice broker's methodology aims at minimising the control plane traffic since it minimises the number of active slices. Other approaches, proposed by the literature, are not considered in the analysis of this article. Hence, the numerical evaluations consider the load of control traffic negligible.

Figure 2 shows the two sample topologies, which are used to apply the new proposed model. The incoming end-to-end services are single-source unicast and can have random source and destination, chosen among the vertices of the two topologies. In particular, capacity is expressed in Gb/s, latency in ms, while reliability and priority are pure numbers.

B. Discussion on results

Figure 3 shows the variation of mean link utilisation and mean link utilisation per slice for mesh and CalREN topologies, versus simulation time. The evaluations are performed considering variation of link capacity between 10-50 Gb/s (mesh network) and between 100-500 Gb/s (CalREN network). By comparing Figure 3(a) and Figure 3(b), some differences appear. The former displays that the increase in capacity of backbone links does not change the achievement of 0.8 average link utilisation. However, the latter depicts that augmenting link capacity not only cannot increase the average link utilisation but also it presents inefficient link utilisation, decreasing from 0.6 to about 0.4-0.3.

What is the reason for that behaviour? In existing 4G networks, the main aim is to provide higher and higher capacity to guarantee better quality to services. The analyses in existing literature about slicing had still applied this policy in the context of future generations virtualised networks. Nevertheless, if we consider the presence of different verticals as 5G and B5G networks (as done in this article), the requirements in terms of latency can limit the use of all the 'capacity potentials' of the backbone. In fact, even if demand of capacity is satisfied, latency can deny the acceptance of a service belonging to uMTC or mMTC.

This limitation appears clearer especially when the size of the network increases. In the small mesh network, sources and destinations are closer with high probability than in the CalREN topology, where latency can become a hard threshold on the efficient use of capacity resources.

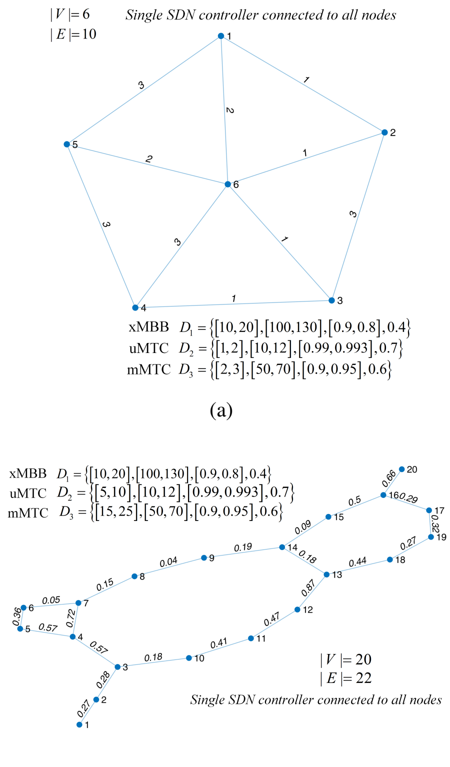

Fig. 2. Sample networks topologies used in the paper. The labels of the edges refer to the values of propagation delay (a) Small mesh network (b) CalREN-like real topology. The values of propagation delay are calculated using physical distances between real cities, where nodes are located (using Google maps).

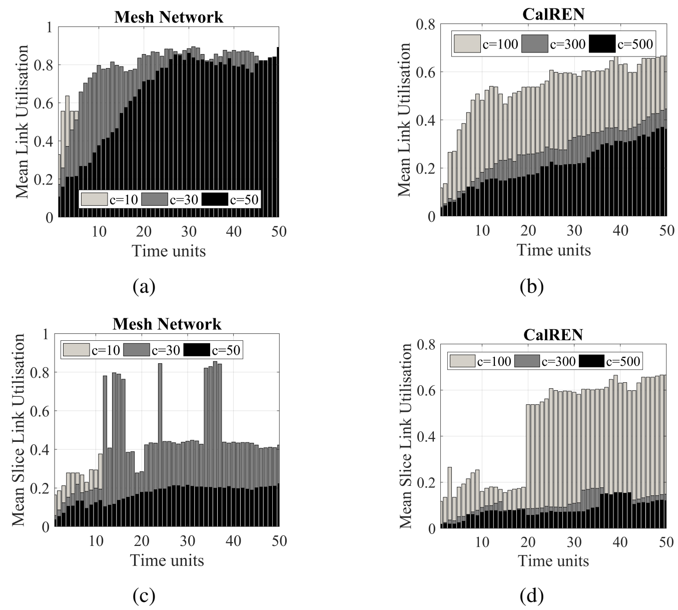

Fig. 3. (a) Mean link utilisation in the mesh network of Figure 2(a) (b) Mean link utilisation in the CalREN-like network of Figure 2(b) (c) Mean link utilisation per slice in the mesh network (d) Mean link utilisation per slice in the CalREN-like network.

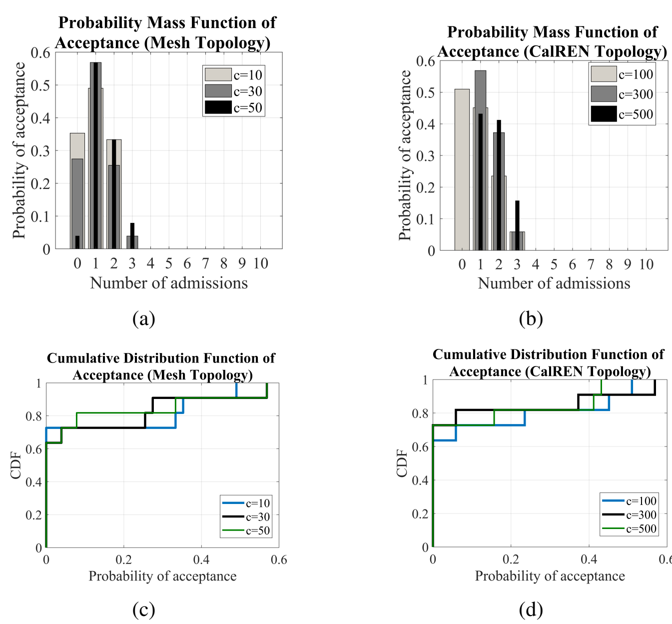

Fig. 4. (a) Probability mass function of end-to-end service admissions for the mesh network (b) Probability mass function of end-to-end service admissions for the CalREN-like topology (c) Empirical cumulative distribution function of probability distribution of number of service admissions in the mesh network (d) Empirical cumulative distribution function of probability distribution of number of service admissions in the CalREN-like network.

Figure 3(c) and Figure 3(d) confirm the above considerations, per each slice. The respective variation of the number of slices versus simulation time is depicted in Figure 5. Obviously, an increase in the number of slices reduces the average link utilisation per slice.

Figure 4 illustrates the probability mass function (pmf) and the cumulative distribution function (CDF) of end-to-end service admissions, for both mesh and CalREN-like network. Figure 4(a) and Figure 4(b) enforces and complete previous deductions with additional information. The probability of a service to be accepted is higher in case of xMBB services. On the other hand, when more demanding services in terms of latency (mMTC and especially uMTC) are asking for virtual resources, the probability to be accepted decreases significantly. In particular, this behaviour remains independent of the increasing provisioned link capacity.

Next, Figure 4(c) and Figure 4(d) highlight how the CDFs in both scenarios is generally independent of link capacity variation.

Finally, Figure 5 connects algebraic-geometric properties of the virtual networks with network properties: in particular, it depicts a strong relationship between the number of active slices and the gcd polynomial of the affine variety of the virtual network. In general, when the number of

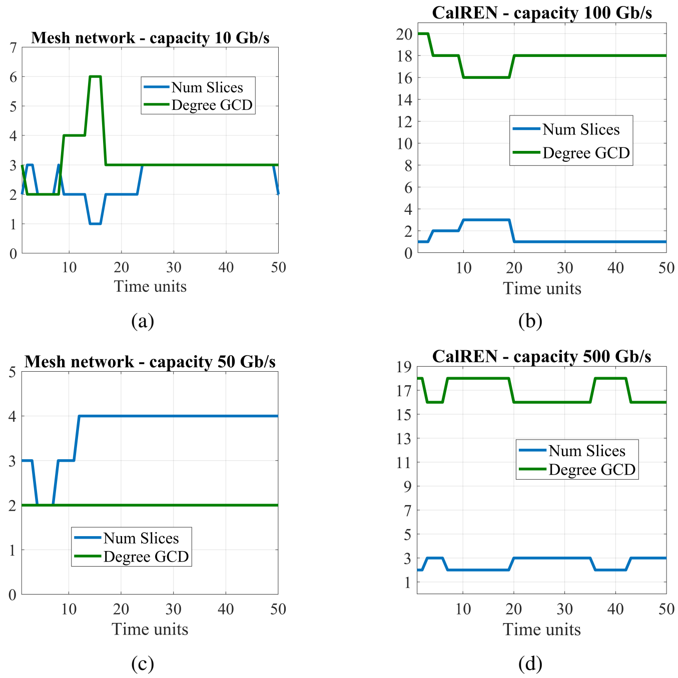

Fig. 5. Number of active slices compared to the variation of degree of gcd polynomial (a) Small mesh network with link capacity 10 Gb/s (b) CalREN-like network with link capacity 100 Gb/s (c) Small mesh network with link capacity 50 Gb/s (d) CalREN-like network with link capacity 500 Gb/s.

slices increase/decrease the degree of gcd polynomial inversely decrease/increase. That relationship is weaker in the small mesh network (Figure 5(a) and Figure 5(c)) but it becomes stronger in the CalREN-like topology (Figure 5(b) and Figure 5(d)), with higher number of edges and nodes. Next, it is important to notice that the ranges of values assumed by the degree of gcd polynomials and the number of slices in the networks are independent of variations of provisioned link capacity. Moreover, it is possible to define a maximum number of active slices for each specific topology, independent of the capacities and of number of arrivals/departures of services. In this algebraic-geometric context, we want also to remark that both the scenarios set up virtual networks represented by finite varieties and the common solution to all their equations is always 0. Additionally, all the virtual network polynomials are never coprime. As underlined by results, latency is an important requirement to consider in 5G and B5G network design. Moreover, the higher usage of available capacity increase latency and queueing time, thus increasing the rejection of services.

IV. Conclusions

To the best of authors' knowledge, this article has defined a general and rigorous model based on multilayer graph theory, linear algebra and algebraic graph theory, which is capable to better capture the complexity of future 5G and B5G networks. In particular, the new model also takes into account the heterogeneity of demands by verticals: this is represented by demand sets with multiple elements instead of single element (like capacity in 4G). Furthermore, the use of algebraic graph theory in virtual network slicing permits the connection of network slicing and algebraic geometry, opening further horizons in the study of virtual network properties and algorithmic slicing.

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