TECHNISCHE UNIVERSITÄT DRESDEN

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Quantum Communication Networks

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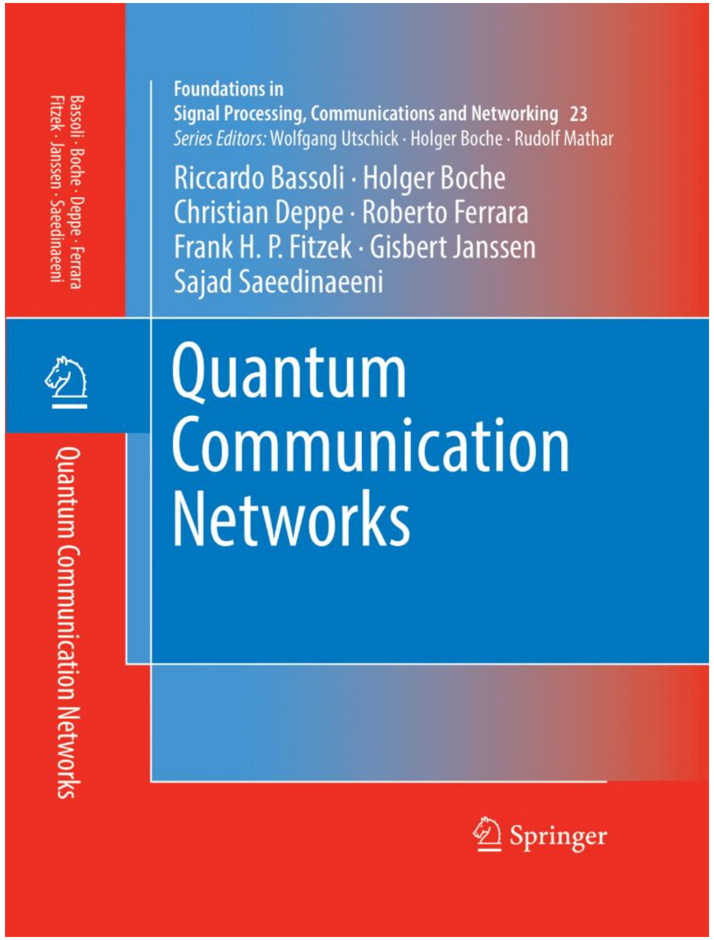
Online, 28.01.2021

Future Classical-Quantum Communication Networks

Published book “Quantum Communication Networks”

(Deutsche Telekom Chair of Communication Networks TUD, TUM-LNT, TUM-LTI)



Learning the Past to Understand Present and Future

W. Gilbert Electric effects and electric states.

C.-F. d. C. Du Fay Two different types of electric charge.

A. Volta Electricity is dynamic. Invention of the pile.

H.C. Oersted, S.D. Poisson, A.-M. Ampère, M. Faraday

Electrodynamics and experiments to reveal electromagnetic effects

1500 | 1600 | 1700 | 1800 | … → Time

S. Gray Electricity can be transmitted via contact, wires or sticks of specific materials (conductors).

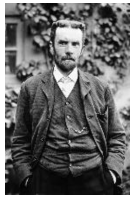
C.-A. de Coulomb Electric action at a distance.

Attraction/repulsion between magnetic poles.

S. T. von Sömmerring

Electric telegraph.

J.C. Maxwell, O. Heaviside



Complete mathematical theory of electromagnetism.

Learning the Past to Understand Present and Future

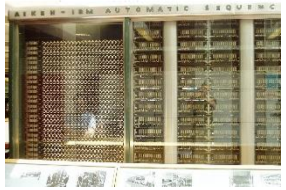
R. Watson-Watt


First Radar (1935).

A. Turing

Turing Machine (1937)

IBM Computer



Harvard Mark I (1940).

Nippon Telegraph and Telephone (1979)

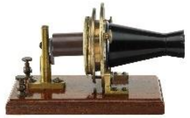
The first commercial automated **wireless cellular network** (1G) analog was launched in Japan.

WWW (1980s)

Robert Cailliau, Jean-François Abramatic, and Tim Berners-Lee

… 1900 2000 Time

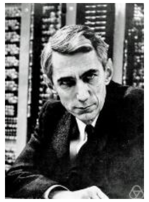
Bell telephone (1876)



G. Marconi


Wireless telegraphy (1896)

C.E. Shannon



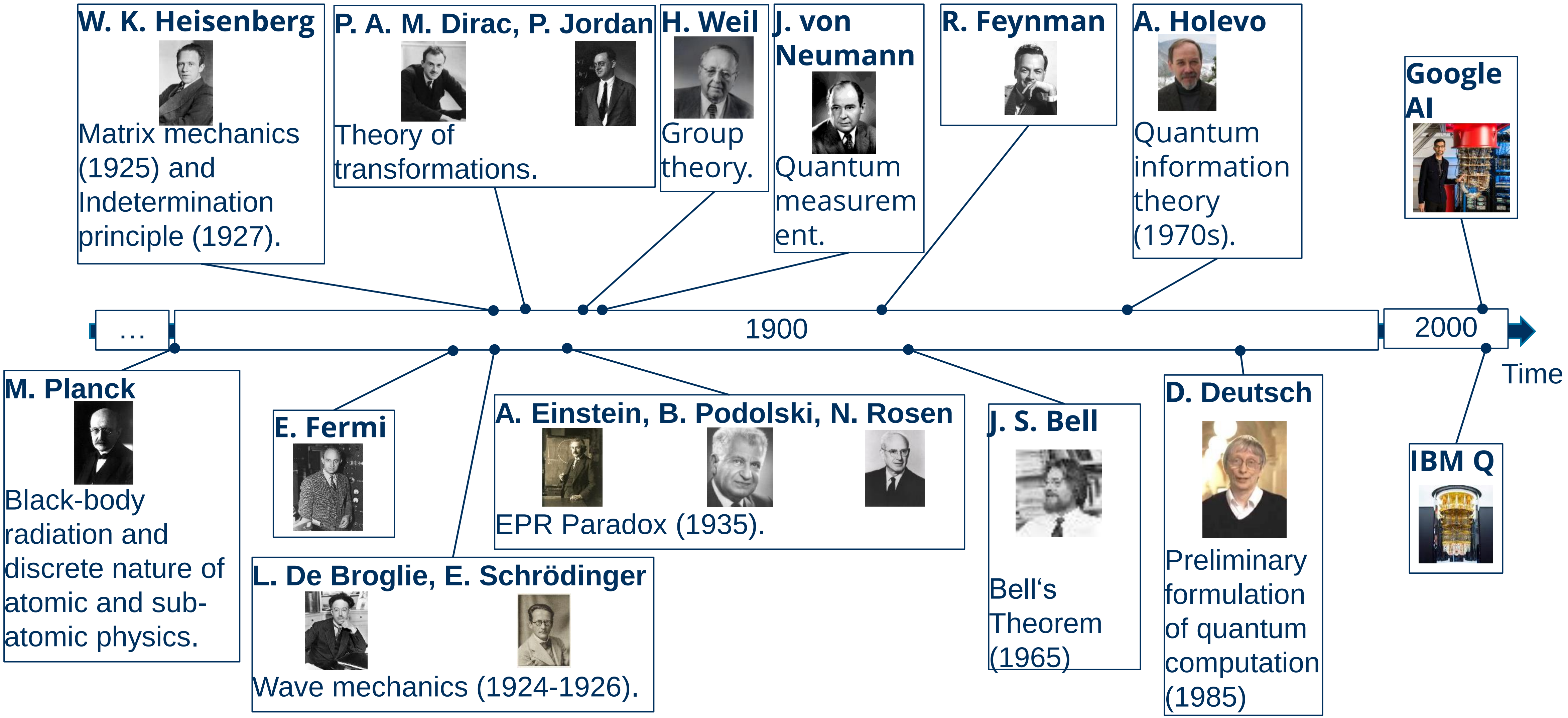
C.E. Information theory (1948)

Mobile phones from 1994 to 2004

lte lte 4DVANCED lte Advanced Pro 5G

Ecosystem of wired-wireless networks, and network softwarization.

Learning the Past to Understand Present and Future



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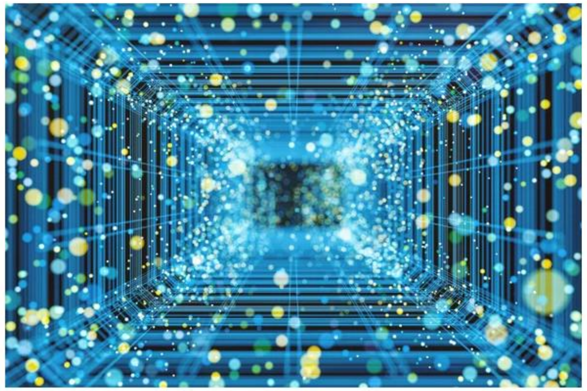
COMPUTING

Google's Quantum Computer Achieves Chemistry Milestone

A downsized version of the company's Sycamore chip performed a record-breaking simulation of a chemical reaction

By Neil Savage on September 4, 2020



Credit: Getty Images

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Last year's quantum superiority experiment was run on a chip dubbed Sycamore, which contained 53 superconducting quantum bits, or qubits. Chilled to near absolute zero, the qubits take on quantum-mechanical properties, allowing scientists to manipulate them in more complicated and useful ways than the simple "on/off" flows of current that make up the bits of classical computers. The hope is that one day, quantum computers will become powerful enough to quickly perform calculations that would take the lifetime of the universe for a classical computer to complete.

https://www.scientificamerican.com/article/googles-quantum-computer-achieves-chemistry-milestone/

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IBM researchers have already installed the mounting hardware for a jumbo cryostat big enough to hold a quantum computer with 1 million qubits. CONNIE ZHOU/IBM

IBM promises 1000-qubit quantum computer—a milestone—by 2023

By Adrian Cho | Sep. 15, 2020 , 5:45 PM

For 20 years scientists and engineers have been saying that "someday" they'll build a full-fledged quantum computer able to perform useful calculations that would overwhelm any conventional supercomputer. But current machines contain just a few dozen quantum bits, or qubits, too few to do anything dazzling. Today, IBM made its aspirations more concrete by publicly announcing a "road map" for the development of its quantum computers, including the ambitious goal of building one containing 1000 qubits by 2023. IBM's current largest quantum computer, revealed this month, contains 65 qubits.

"We're very excited," says Prineha Narang, co-founder and chief technology officer of Aliro Quantum, a startup that specializes in code that helps higher level software efficiently run on different quantum computers. "We didn't know the specific milestones and numbers that they've announced," she says. The plan includes building intermediate-size machines of 127 and 433 qubits in 2021 and 2022, respectively, and envisions following up with a million-qubit machine at some unspecified date. Dario Gil, IBM's director of research, says he is confident his team can keep to the schedule. "A road map is more than a plan and a PowerPoint presentation," he says. "It's execution."

IBM is not the only company with a road map to build a full-fledged quantum computer—a machine that would take advantage of the strange rules of quantum mechanics to breeze through certain computations that just overwhelm conventional computers. At least in terms of public relations, IBM has been playing catch-up to Google, which 1 year ago grabbed headlines when the company announced its researchers had used their 53-qubit quantum computer to solve a particular abstract problem that they claimed would overwhelm any conventional computer—**reaching a milestone known as quantum supremacy**. Google has its own plan to build a million-qubit quantum computer within 10 years, as Hartmut Neven, who leads Google's quantum computing effort, explained in an April interview, although he declined to reveal a specific timeline for

https://www.sciencemag.org/news/2020/09/ibm-promises-1000-qubit-quantum-computer-milestone-2023

Important Initial Remarks

1. “**Microcosmos is not visualisable**” (regarding the intention to represent subatomic particles via classical models such as spheres or other bodies. In this way, we assign geometric and cinematic properties that does not exist in the subatomic scale).

   Quantum mechanics logically change the mentality of having a visual representation of the physical system.

2. **Interaction with the system changes its characteristics.**

Classical

Quantum

The Beginning of the Story

The Origins of Quantum Mechanics – Experiment 1

Quantum mechanics (ger. *Quantenmechanik*) studies the behaviour of systems on the atomic and subatomic scale.

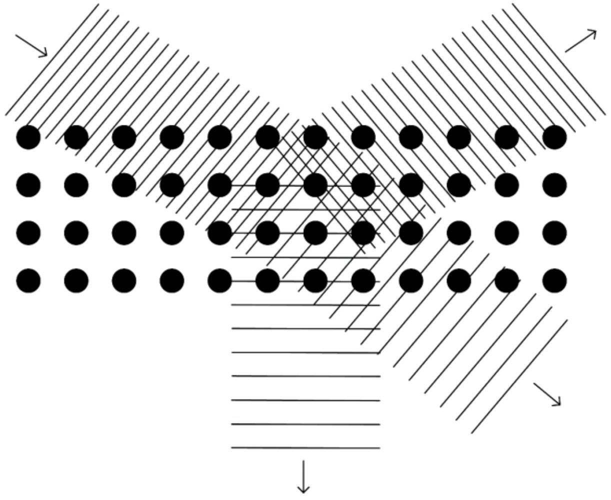
All atoms have approximately the same size, which is independent of the number of electrons. The radius of an atom is generally 1 Å to 2Å (ångström). The ångström is defined as 10^{-10} m.

1927 – C. J. Davisson and L. H. Germer studied the reflections and scattering of electrons into crystals.

The ingoing and outgoing electrons with an initial 3-momentum are reflected or deflected in the crystal according to different specific angles. This shows a **periodic (wave) inherent behaviour of electrons**.

Given a general particle of momentum *p*, a relation similar to the one existing for waves exists, so that

$$\lambda = \frac{h}{p} = \frac{2\pi\hbar}{p}$$



where *λ* is the wavelength and *h* is the Planck's constant.

The *λ* associated to a particle of momentum *p* is called its **de Broglie wavelength**, (revealed in 1923).

According to Relativity, a particle has a frequency, given in terms of its energy (**Planck's formula**) such that

$$E = h\nu = 2\pi\hbar\nu$$

The Origins of Quantum Mechanics – Experiment 2

Another part of the research community was thinking the other way around. In fact, theories were proposed to think about Maxwell's electromagnetic waves as having inherent nature of particles.

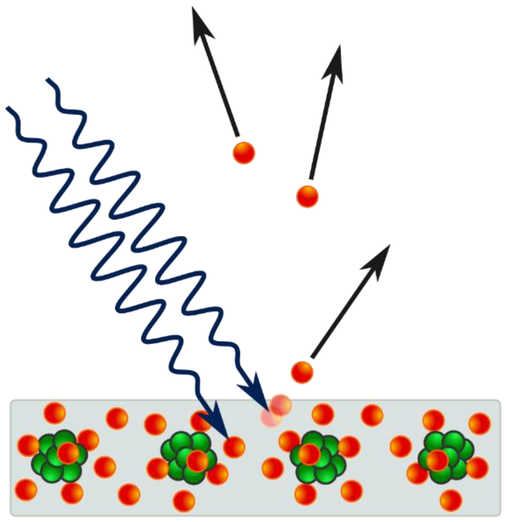
Photoelectric effect – firstly observed by H. Hertz (1887) and P. Lenard (1902), and finally explained by A. Einstein (1905), for which he won the Nobel prize in 1921.

The photoelectric effect happens when light with specific frequency reaches a metal, producing electrons. The production of electrons does not depend on the intensity of the light.

Einstein described this phenomenon considering the electromagnetic radiation composed of particles (called **photons**) with the energy described by the Planck's formula

$$E = h\nu = 2\pi\hbar\nu$$

The emitted electrons are produced when photons hit atoms with specific frequencies.



Photoelectric effect. In Wikipedia, The Free Encyclopedia. Retrieved from https://en.wikipedia.org/w/index.php?title=Photoelectric_effect&oldid=990200940

The Origins of Quantum Mechanics – Experiment 3

However, the first ’quantum’ characteristics of particles were revealed by M. Planck (**1900**) during his analysis of **black-body radiation**,

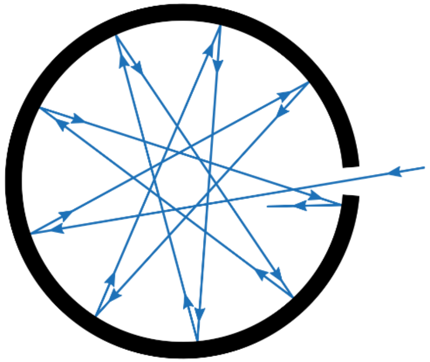
He investigated the electromagnetic radiation in equilibrium with its ‘black’ material surroundings, all kept at some specific temperature *T*. The specific intensity *I* is expressed by

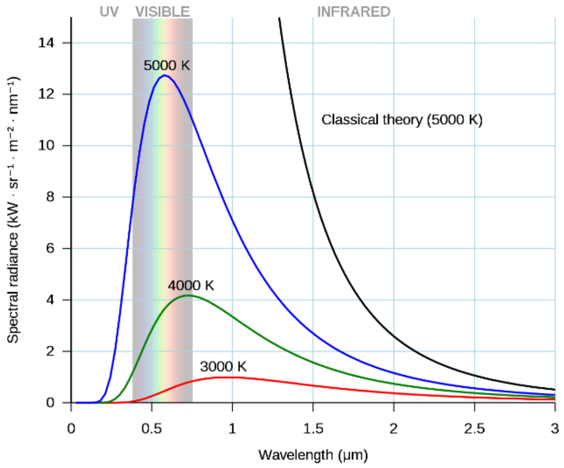
$$I = \frac{2h\nu^3}{e^{\frac{h\nu}{kT}} - 1}$$

k is the **Boltzmann’s constant**.

Planck derived it after a statistical analysis of many experimental measurements, so that he realised that the electromagnetic waves could only be absorbed or emitted in bundles of defined energy values *E*, directly related to the frequency of the wave

$$E = h\nu = 2\pi\hbar\nu$$





Wikipedia contributors. (2020, November 3). Black body. In Wikipedia, The Free Encyclopedia. Retrieved from https://en.wikipedia.org/w/index.php?title=Black_body&oldid=986848461

The Origins of Quantum Mechanics – Experiment 3

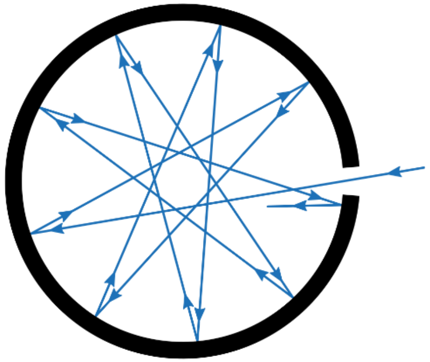
He also calculated the value of the constant *h* to be 6.62×10^{-34} Js (the so-called **Planck's constant**).

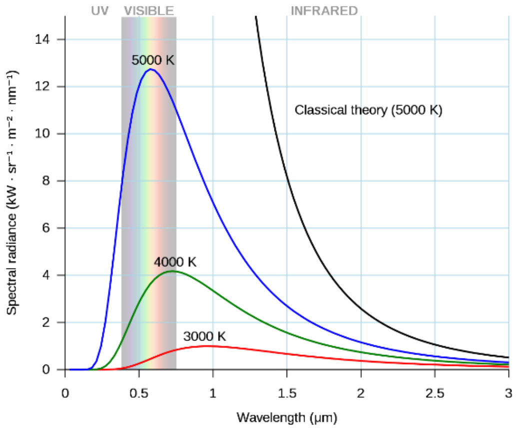
Such bundles were later referred to specific particles (and not electromagnetic oscillations), called **photons**.

The use of the Dirac's form $2\pi\hbar$ of *h* helps to find the subsequent logic connection between the wavelength and the momentum

$$\nu^{-1} = \lambda = \frac{2\pi\hbar}{p} \qquad E = h\nu = 2\pi\hbar\nu$$

Then, it appears that both energy and momentum are linked to a period, which is temporal for the former and spatial for the latter.





Wikipedia contributors. (2020, November 3). Black body. In Wikipedia, The Free Encyclopedia. Retrieved from https://en.wikipedia.org/w/index.php?title=Black_body&oldid=986848461

Quantum Mechanics – Postulates

Postulate 1. The states of quantum systems can be described via vectors in a complex Hilbert space. A linear decomposition into vector states is also called a superposition of vector states. In particular, given a classical system, its quantization will assign a vector to each classical state and the space of quantum states will be a Hilbert space in this vector space.

Postulate 2. The evolution in time of a closed (isolated) quantum system follows Schrödinger's equation, which implies that the evolution of the quantum state follows a unitary evolution **U**.

Postulate 3. A physical observable is a Hermitian operator. Measuring an observable yields one of its possible eigenvalues, with the probability given by the magnitude of the normalized state vector onto the eigenspace of the outcome. Contrarily to Postulate 2, this causes a loss of natural quantum coherence and the evolution of the quantum system generally implies an irreversible loss of information and the destruction of the superpositions across the eigenspaces (collapse of the wavefunction).

Postulate 4. States of composite quantum systems. The Hilbert space of a composite system is represented by the tensor product of the Hilbert spaces of single subsystems. Similarly, the space of composite observables is the space of Hermitian operators of the tensor product Hilbert space.

Wavefunction, Spin and Qubit

The ‘Reality’ of Quantum Mechanics

In scientific research, there are two main philosophical positions.

1. **Positivist**. Refuses to address the problem of the reality of the quantum-mechanical description. It only uses the quantum-mechanical formalism to describe the physical system in accordance to the experiments.
2. There is some **’reality**’ in the **state vector** (or **wavefunction**). It is important to find a ’concept of reality’ in the theory of quantum mechanics, otherwise all science becomes just a description of what we subjectively see.

By again looking at

$$\psi(x_a) := \mathrm{e}^{-i\frac{\tilde{p}_a x_a}{\hbar}}$$

it is possible to see that the momentum of the particle is not spatially localised but it is uniformly distributed in all the space. In fact, its value (measured via its module) is always 1.

What has happened to the concept of a particle with defined momentum, and spatial location and direction?

The Phase of the Wavefunction

The **wavefunction** is not a wave of real values, so its variation does not necessarily come from the variation of its intensity (or amplitude).

The quantum wavefunction has complex values and its oscillation is a variation of the state of the momentum in

$$-\tilde{p}_a x_a/\hbar$$

measured on the unit circle in the complex plane.

This argument of the wavefunction is usually called **phase**. And such phase is not a ’wave’ but mainly rotating on the unit circle.

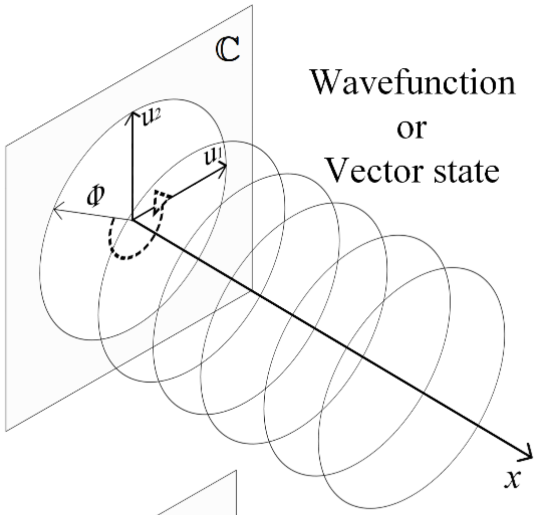
Classical Versus Quantum ‘Wave’

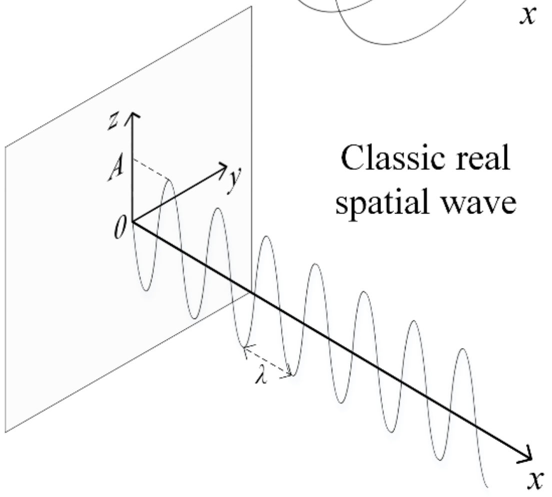
Classical wave motion is defined as a propagation of disturbances – that is, deviations from a state of rest or equilibrium – from place to place in a regular and organized way.

Quantum wavefunction is produced by the rotation of ψ on the complex plane, along all the three spatial directions (x, y and z).

Directions u_1 and u_2 are not ordinary spatial directions but they only represents the complex plane of all the possible values ψ can have.

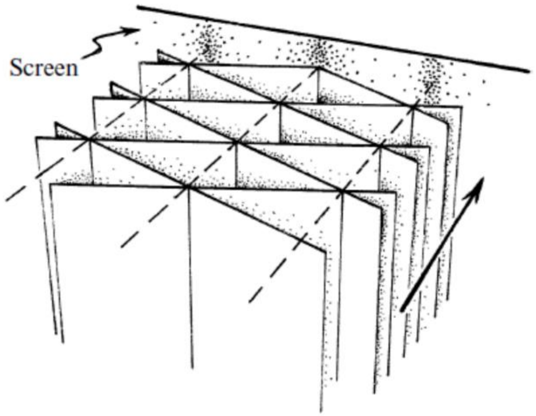
The total number of dimensions is 5.





Classical Versus Quantum ‘Wave’

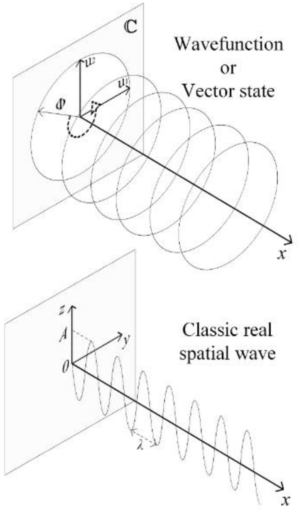
It is still not clear how the wavefunction ψ can also look like a particle (as in the two-slit experiment).



The logical problem is that we should not think the wavefunction as a classical wave, providing local disturbances.

In a classical wave what happens in a specific point is independent of the other ones. On the other hand, the fundamental characteristic of ψ is its **nonlocality**, then what happens in a point affects all the other points of the wavefunction.

In this sense, the wavefunctions are **holistic entities**. This behaviour of the wavefunction theoretically happens at any distance.



R. Penrose. The Road to Reality: A Complete Guide to the Laws of the Universe. Vintage Series. Vintage Books, 2007.

Classical Versus Quantum ‘Wave’

If a photon is sent through a beam-splitter and it goes to two different widely-separated parts (even light-years distance), in the moment one reveals the photon, the other one immediately becomes unable to see it. The above considerations shows that ψ becomes a localised entity only at the moment of measurement, and this localisation happens independently of how much the wavefunction has previously been dispersed.

The particle again evolves according to Schrödinger’s equation as a nonlocal entity. These ’jumps’ given by the dualism wave-particle are described by the **unitary evolution** (wavefunction in Schrödinger’s equation) and the **quantum measurement** (particle).

In **classical physics** these two aspects behave the same, in quantum mechanics they are completely different.

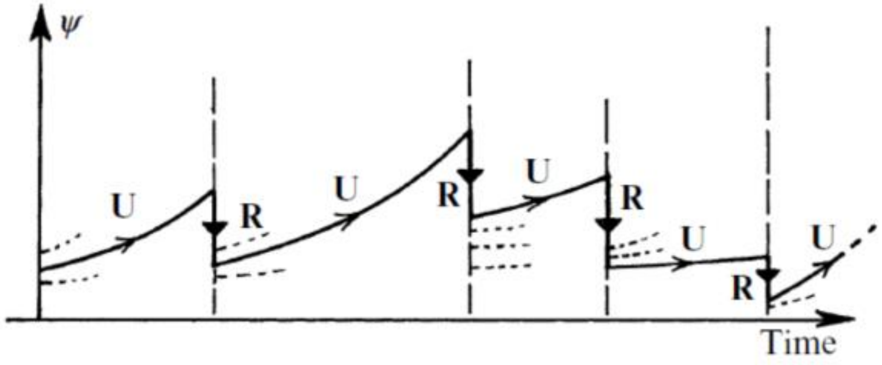
Schrödinger's Equation and Measurement

A single particle (not-relativistic) is described by a **state vector** or **wavefunction**, and its evolution in time can be expressed by **the Schrödinger's equation**.

The measurement corresponds to an operator – usually a Hermitian operator –, and the result of this process is the state moving to an eigenstate of this operator.

The choice of the specific eigenstate is a matter of probability.

This jump of the operator to one of its eigenstates is called **state-vector reduction** or **collapse of the wavefunction**. Immediately after the measurement, the Schrödinger's evolution starts again.



Let's denote Schrödinger's evolution by **U** and state reduction by **R**.

The use of the letter **U** stands for unitary evolution. In a sense Schrödinger's equation is indeed 'unitary'.

it preserves the scalar product $\langle\psi|\varphi\rangle$ between two elements $|\psi\rangle$,$|\varphi\rangle$ of H. That is to say, $\langle\psi|\varphi\rangle$ is constant in time $\frac{d}{dt}\langle\psi|\phi\rangle = 0$

R. Penrose. The Road to Reality: A Complete Guide to the Laws of the Universe. Vintage Series. Vintage Books, 2007.

Linearity of the Schrödinger's Equation

Let us examine the Schrödinger's equation

$$i\hbar\frac{\partial\psi}{\partial t} = \mathrm{H}\psi$$

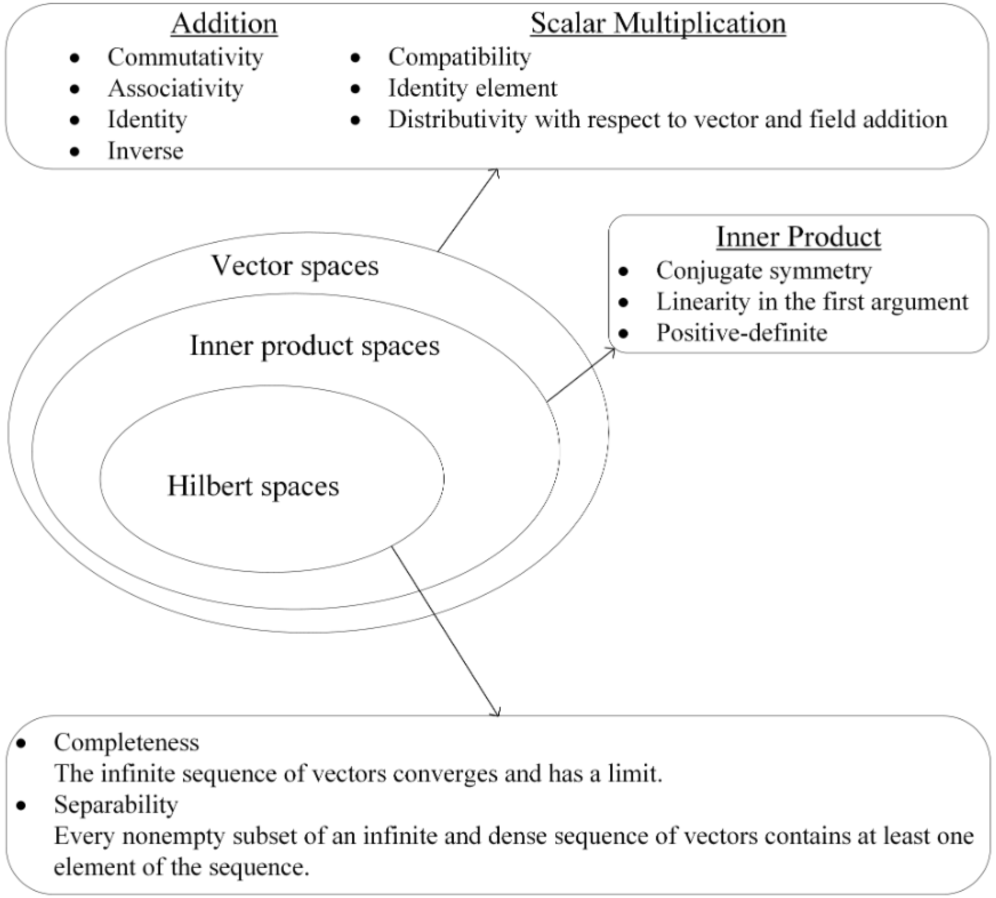
We shall imagine the Hamiltonian H to be known.

We must note:

1. It is a deterministic equation (the time-evolution being completely fixed once the state is known at any one time).
2. The Schrödinger's equation is a complex equation, owing to the manifest appearance of *i* on the left (and there are many more possibilities for occurrence of *i* in the Hamiltonian).
3. The Schrödinger's equation is indeed linear, in the sense that if *ψ* and *φ* are solutions (with the same H) then so also is any linear combination *wψ+zφ*, where *w* and *z* are complex constants. By adding *w* times the first of the above equations to *z* times the second we get

$$i\hbar\frac{\partial}{\partial t}(w\psi + z\phi) = \mathrm{H}(w\psi + z\phi)$$

Wavefunctions and the Hilbert Space



R. Bassoli, H. Boche, C. Deppe, R. Ferrara, F. H. P. Fitzek, G. Janssen, S. Saeedinaeen, "Quantum Communication Networks", 1st Ed., Springer, 2021, ISBN: 978-3-030-62938-0.

Dirac’s Notation

Bras and kets are elements of dual vector spaces, however the spaces are different and thus bras and kets cannot be summed together.

In spaces with a scalar/inner product (a complex number in complex vector spaces), the inner product between two vectors can be seen as simply the action of one of the dual vectors onto the remaining vector.

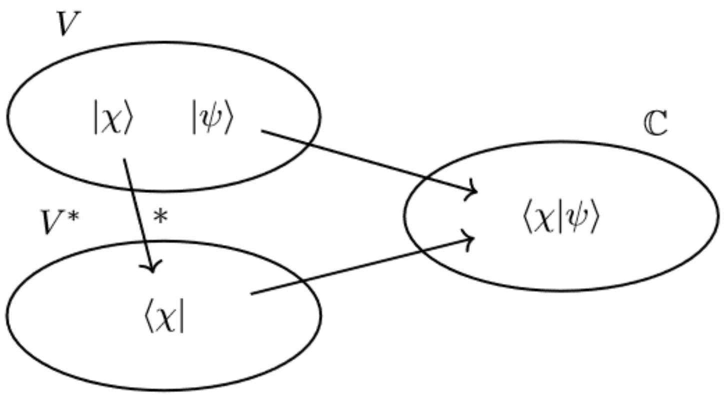
In Dirac’s notation, the **inner product** is denoted by $\langle \chi | \psi \rangle \in \mathbb{C}$

which makes it seamless in interpreting it as an inner product, as the action of a bra on a ket, or as the action of a ket on a bra. More explicitly, it is important to note that $\langle \chi | = | \chi \rangle^* \in H^*$

In complex spaces it holds that $\langle \chi | \psi \rangle = \langle \psi | \chi \rangle^*$

Two vectors are orthogonal if their inner product is zero and in Dirac’s notation for two kets $|\chi\rangle$,$|\psi\rangle$, the condition is thus written as $\langle \chi | \psi \rangle = 0.$

Then, two quantum states are said to be orthogonal if the corresponding vectors are orthogonal.



$\lvert\cdot\rangle + \lvert\cdot\rangle$	sum of kets	→ complex vector
$\langle\cdot\rvert + \langle\cdot\rvert$	sum of bras	→ complex vector
$\langle\cdot\lvert\cdot\rangle$	scalar product (braket)	→ complex number
$\lvert\cdot\rangle\langle\cdot\rvert$	outer product	→ complex matrix

R. Bassoli, H. Boche, C. Deppe, R. Ferrara, F. H. P. Fitzek, G. Janssen, S. Saeedinaeen, “Quantum Communication Networks”, 1st Ed., Springer, 2021, ISBN: 978-3-030-62938-0.

Dirac's Notation

In Dirac's framework, a complex *n*-dimensional vector is defined as

$$|\psi\rangle = \sum_{i=1}^{n} c_i |u_i\rangle$$

where coefficients $c_1, \dots, c_n$ are complex numbers, which multiply a general basis $|u_1\rangle, \dots, |u_n\rangle$.

In quantum theory the coefficients $c_1, \dots, c_n$ are also called **amplitudes**, due to their role in expectation values.

This is the so-called **Dirac's bra-ket** notation or **bra-ket notation**.

In this notation, state vectors are denoted by $|\ \rangle \in H$, also called **kets**, in which there is any suitable label indicating the state in question.

The Spin of a Particle

Classical physics. Given an object in motion, we call momentum the product between its mass (scalar) and its velocity (vector).

Relativity. There is a proportionality between energy and mass ($E = mc^2$), so the concept of momentum could be extended to any form of energy propagating in the space.

Quantum Mechanics. The concept of 'quantization' led during the 1920s to the development of quantum mechanics, which appeared to provide physicists with the correct method of calculating the structure of the atom. In his model N. Bohr had postulated that the electrons in the atom move only in orbits in which the angular momentum (angular velocity multiplied by mass) has certain fixed values. Each of these allowed values is characterised by a quantum number that can have only integer values.

In the full quantum mechanical treatment of the structure of the atom, developed in the 1920s, three quantum numbers relating to angular momentum arise because there are three independent variable parameters in the equation describing the motion of atomic electrons.

The Spin of a Particle

In 1925, however, two Dutch physicists, S. Goudsmit and G. Uhlenbeck, realised that, in order to explain fully the spectra of light emitted by the atoms of alkali metals, such as sodium, which have one outer valence electron beyond the main core, there must be **a fourth quantum number** that can take only two values, $-\frac{1}{2}$ and $+\frac{1}{2}$.

Goudsmit and Uhlenbeck proposed that this quantum number refers to an internal angular momentum, or **spin**, that the electrons possess.

This implies that the electrons, in effect, behave like spinning electric charges. Each therefore creates a magnetic field and has its own magnetic moment.

The internal magnet of an atomic electron orients itself in one of two directions with respect to the magnetic field created by the rest of the atom.

It is either parallel or antiparallel; hence, there are two 'quantized' states – and two possible values of the associated spin quantum number. Now, the concept of spin is recognised as an intrinsic property of all sub-atomic particles. Indeed, spin is one of the key criteria used to classify particles into two main groups:

- fermions, with half-integer values of spin ($\frac{1}{2}, \frac{3}{2}, \ldots$) (all of the 'matter' particles, e.g. quarks and leptons),
- bosons, with integer values of spin (0,1,2,...) ('force' particles such as photons).

The Spin of a Particle

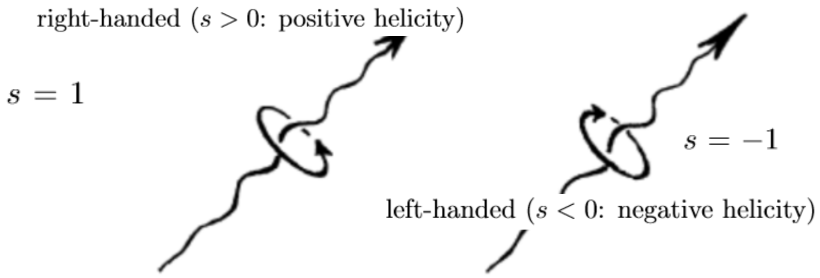
The introduction of the spin requires some changes in the quantum formalism. In the non-relativistic case, a particle having spin s replaces the wavefunction $\psi(\mathbf{x},t)$ with $\psi(\mathbf{x},s,t)$.

Since this spin s can only assume two values, $\psi^+(\mathbf{x},t)$ and $\psi^-(\mathbf{x},t)$, the squared value of the norm gives the density of probability to find the particle (at a specific time t) in the position $\mathbf{x}$ with parallel or antiparallel spin.

In the case of a massive particle, the spin is the angular momentum about its centre of mass.

In the case of a massless particle (e.g. photon), the spin behaves in a way that is a little different from the more usual spin of a massive particle (e.g. an electron or proton). We must think of a photon (or other massless particle) as necessarily spinning around its direction of motion.

Additionally, the spin state can be any (quantum) linear combination of the two (simply linear combinations of the right-and left-handed states).



R. Penrose. The Road to Reality: A Complete Guide to the Laws of the Universe. Vintage Series. Vintage Books, 2007.

Group Theory, Angular Momentum and Pauli Matrices

As with practically everything else in quantum mechanics, the angular momentum components $\mathbf{L}_1, \mathbf{L}_2, \mathbf{L}_3$ must act as linear operators on the Hilbert space H.

Thus, quantum systems possessing angular momentum provide a representation of the Lie algebra of SO(3) in terms of linear transformations of H.

In particular, the matrices

$$\mathbf{L}_1 = \frac{\hbar}{2}\begin{pmatrix} 0 & 1 \\ 1 & 0 \end{pmatrix}$$

$$\mathbf{L}_2 = \frac{\hbar}{2}\begin{pmatrix} 0 & -i \\ i & 0 \end{pmatrix}$$

$$\mathbf{L}_3 = \frac{\hbar}{2}\begin{pmatrix} 1 & 0 \\ 0 & -1 \end{pmatrix}$$

called (without the $\frac{\hbar}{2}$) the **Pauli matrices**, provide the simplest (non-trivial) representation of angular momentum.

The Geometry of Spin

We can arrange spin up $|\uparrow\rangle$ (right-handed about the upward vertical) to be spin state {1,0};

spin down $|\downarrow\rangle$ (right-handed about the downward vertical) is to be {0,1}.

These two basis states are orthogonal:

$$\langle\uparrow|\downarrow\rangle = 0.$$

We also normalise:

$$\langle\uparrow|\uparrow\rangle = 1 = \langle\downarrow|\downarrow\rangle$$

The general spin-$\frac{1}{2}$ state $\psi=\{c_1, c_2\}$ (general element of H^2), is the linear combination

$$|\psi\rangle = \{c_1, c_2\} = c_1|\uparrow\rangle + c_2|\downarrow\rangle$$

$$|\psi\rangle = \sum_{i=1}^{n} c_i|u_i\rangle$$

of these two basis states.

The Geometry of Spin

The scalar product of another general state ϕ ={a,b} ($|\phi\rangle = a|\uparrow\rangle + b|\downarrow\rangle$) with $\{c_1, c_2\}$ is given by

$$\langle\phi|\psi\rangle = \langle\{a,b\}|\{c_1,c_2\}\rangle = \bar{a}c_1 + \bar{b}c_2.$$

It now turns out that every spin-$\frac{1}{2}$ state must actually be a pure state of spin that is right-handed about some direction in space, so we can write

$$|\nearrow\rangle = c_1|\uparrow\rangle + c_2|\downarrow\rangle$$

where $\nearrow$ is some actual direction in space.

The Geometry of Spin

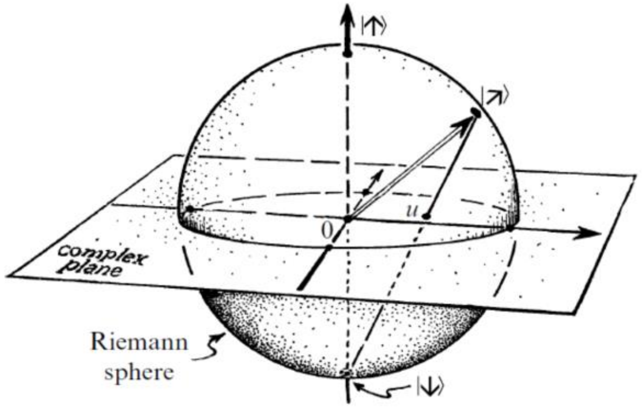
This gives us a remarkable identification between the projective space $\wp\mathrm{H}^2$ and the geometry of directions in space, these directions being thought of as spin directions.

The physically distinct spin-$\frac{1}{2}$ states are indeed provided by this projective space, the different points of $\wp\mathrm{H}^2$ being labelled by the distinct ratios c_1/c_2.

In other words, $\wp\mathrm{H}^2$ is just a copy of the Riemann sphere. Each point of this Riemann sphere labels a distinct spin-$\frac{1}{2}$ state, this being the m=$\frac{1}{2}$ eigenstate of the particular spin measurement that is taken in the direction out to this point, from the centre of the sphere.

We see this geometrical relationship more explicitly if we use the stereographic projection of the sphere from its south pole to its equatorial plane described. This plane is to be regarded as the complex plane of the ratio u=c_2/c_1 of quantum-mechanical amplitudes c_2 and c_1.

This relates the particular point on the sphere, corresponding to the spatial direction ↗, directly to the ratio c_2/c_1.



R. Penrose. The Road to Reality: A Complete Guide to the Laws of the Universe. Vintage Series. Vintage Books, 2007.

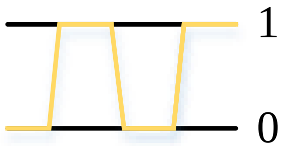
Quantum Bit (Qubit) Definition

"[…] A traditional digital computer employs binary digits, or bits, that can be in one of two states, represented as 0 and 1; thus, for example, a 4-bit computer register can hold any one of 16 (2^4) possible numbers.

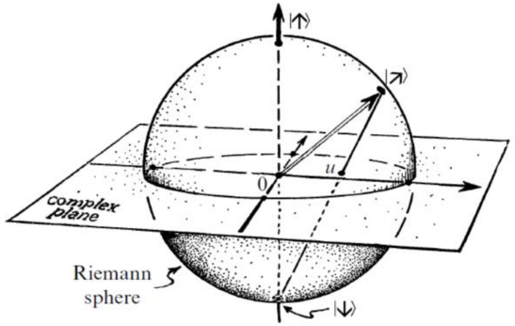
In contrast, a quantum bit (qubit) exists in a **wavelike** superposition of values from 0 to 1; thus, for example, a 4-qubit computer register can hold 16 different numbers simultaneously. In theory, a quantum computer can therefore operate on a great many values in parallel, so that a 30-qubit quantum computer would be comparable to a digital computer capable of performing 10 trillion floating-point operations per second (TFLOPS) – comparable to the speed of the fastest supercomputers. […]"

XOR (sum) (= *b*)

AND (product) (= *b*)



Classical bit
$b \in \mathbb{F}_2$



$$|\psi\rangle = c_1 |\uparrow\rangle + c_2 |\downarrow\rangle$$

$\lvert\cdot\rangle + \lvert\cdot\rangle$	sum of kets →	complex vector
$\langle\cdot\rvert + \langle\cdot\rvert$	sum of bras →	complex vector
$\langle\cdot\lvert\cdot\rangle$	scalar product (braket) →	complex number
$\lvert\cdot\rangle\langle\cdot\rvert$	outer product →	complex matrix

William Coffeen Holton, Quantum computer, Encyclopædia Britannica, August 16, 2020, https://www.britannica.com/technology/quantum-computer

R. Bassoli, H. Boche, C. Deppe, R. Ferrara, F. H. P. Fitzek, G. Janssen, S. Saeedinaeen, "Quantum Communication Networks", 1st Ed., Springer, 2021, ISBN: 978-3-030-62938-0.

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Entanglement

Entanglement – Introduction

Dirac’s notation is general enough to describe **quantum systems with more than one particle.**

A system of more than one particle must be treated as a single holistic unity.

The quantum-Hamiltonian approach, which provides us with the Schrödinger’s equation for the evolution of the quantum state vector,

$$i\hbar\frac{\partial\psi}{\partial t} = \mathrm{H}\psi$$

still applies when there are many particles, possibly interacting. All we need is a suitable Hamiltonian to incorporate all the features.

We do not have a separate wavefunction for each particle; instead, we have one state vector, which describes the entire system.

This single state vector can still be thought of as a wavefunction Ψ.

This Ψ would be a function of all the position coordinates of all the particles.

Entanglement – The Hilbert Space

How do the requirements imposed by the structure of Ψ affect the size of the Hilbert space?

Quantum system composed of *n* particles.

The wavefunction Ψ is in the 3-dimensional space (*t* considered constant) and it is a complex function of 3*n* real variables. This implies $\infty^{2\infty^{3n}}$ degrees of freedom.

_n_ Independent wavefunctions – Toy example.

System composed of 3 independent particles.

The particles can randomly choose among 20 possible states.

Hilbert space has 60 dimensions (H^{20n}).

***Quantum multi-particle* wavefunction – Toy example.**

Quantum system composed of 3 particles.

The particles can randomly choose among 20 possible states.

Hilbert space has 8000 dimensions (H^{20^n}).

Entanglement – The Hilbert Space

Entanglement represents all this ‘extra information’.

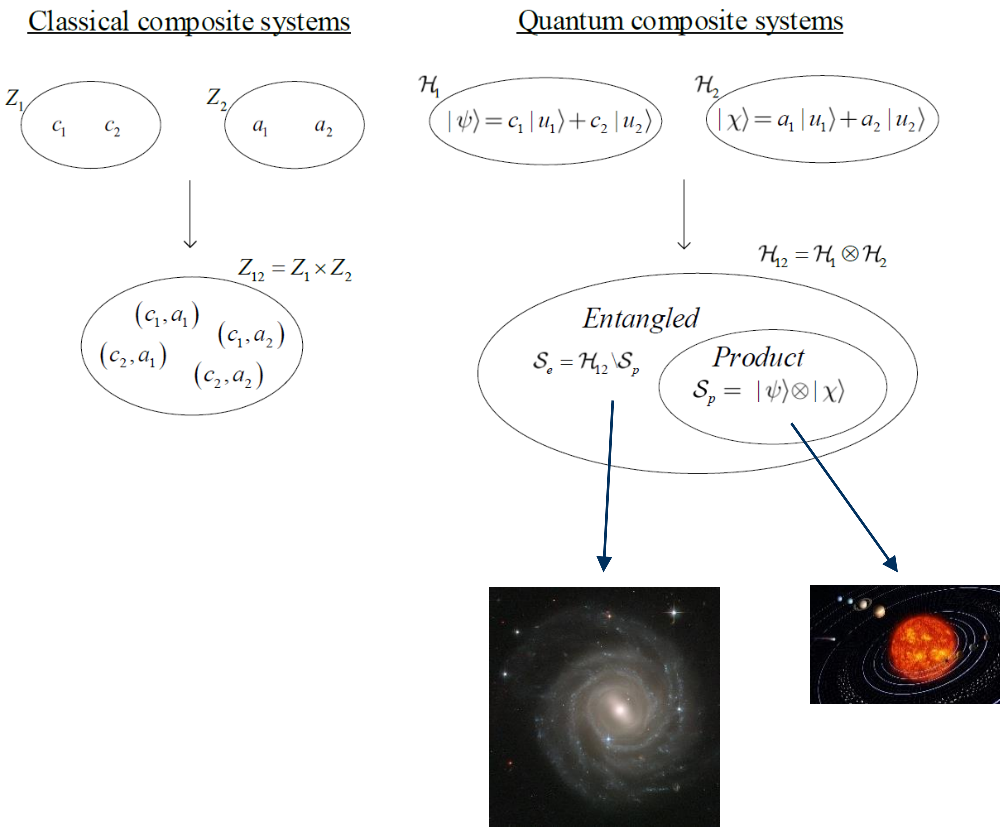
It was originally identified by E. Schrödinger. In fact, he defined it with the German word *Verschränkung* in an article in 1935 ("Die gegenwiirtige Situation in der Quantenmechanik"), where he answered A. Einstein regarding the EinsteinPodolskyRosen (EPR) paradox.

Almost the entire ‘information’ in the wavefunction is concerned with such states! This can tell us something of the potential of quantum computing and networking.

Dirac’s notation

Two particles $|\psi\rangle$ in H^p and $|\chi\rangle$ in H^q.

The quantum state of $|\psi\rangle$ and $|\chi\rangle$ together is $|\psi\rangle \otimes |\chi\rangle$ in $\mathrm{H}^p \otimes \mathrm{H}^q = \mathrm{H}^{pq}$ (we can say $p = q = \infty$).



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Quantum Measurement

Measurement and Quantum Systems

The formalism of the process **R**.

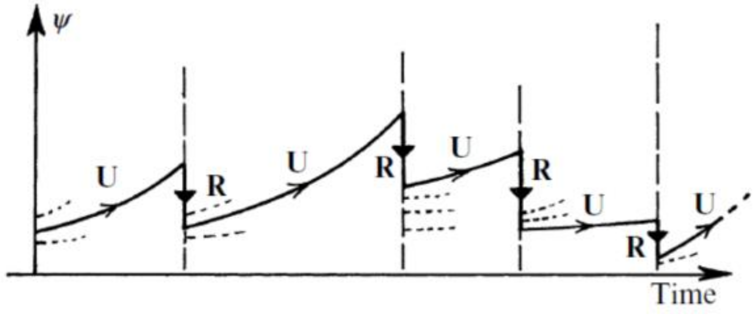
As noted previously, regarding measurements of position and momentum are illustrative examples of what happens in the general case of a quantum measurement.

Some measurable quality of a quantum system would be represented by a certain kind of operator **Q**, called an **observable**, and this operator could be applied to the quantum state. The dynamical variables (e.g. position or momentum) would be examples of observables.

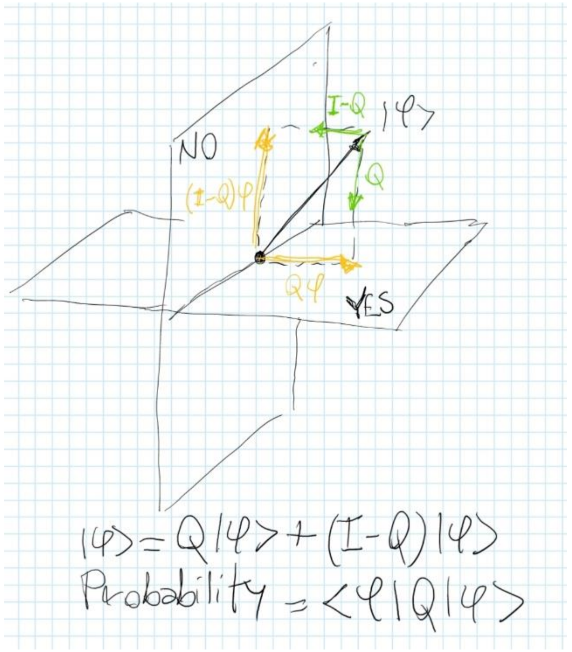
The state of the quantum system before measurement, it jumps to <u>one of the eigenstates of **Q**</u> just as the state is measured, in accordance with **R**. After the measurement, the state acquires a definite value for the observable **Q**, namely the corresponding eigenvalue *q*.

Thus, for each of the different possible results of the measurement of the observable **Q** – that is, for each different eigenvalue $q_1, q_2, \ldots$ – we get one of a set of alternative resulting states, <u>all of which are mutually orthogonal</u>.

R. Penrose. The Road to Reality: A Complete Guide to the Laws of the Universe. Vintage Series. Vintage Books, 2007.



<u>'Is *q* the result of the **Q** measurement?'</u>



Quantum Observable

We see that, indeed, a state never jumps directly to an orthogonal state, in a measurement, because $\langle\psi|\varphi\rangle = 0$ implies that the probability for this would be zero.

In a quantum superposition between orthogonal normalised states ψ and φ, say $c_1|\psi\rangle + c_2|\phi\rangle$

the complex-number weighting factors, c_1 and c_2, are some-times called **amplitudes** – or **probability amplitudes**.

In this case, an experiment set up to distinguish ψ from φ in the state $c_1|\psi\rangle + c_2|\phi\rangle$ would get

ψ with probability $\bar{c}_1 c_1 = |c_1|^2$

φ with probability $\bar{c}_2 c_2 = |c_2|^2$

A similar comment applies to superpositions of more than two states (called **qudits**).

Zurek's Model

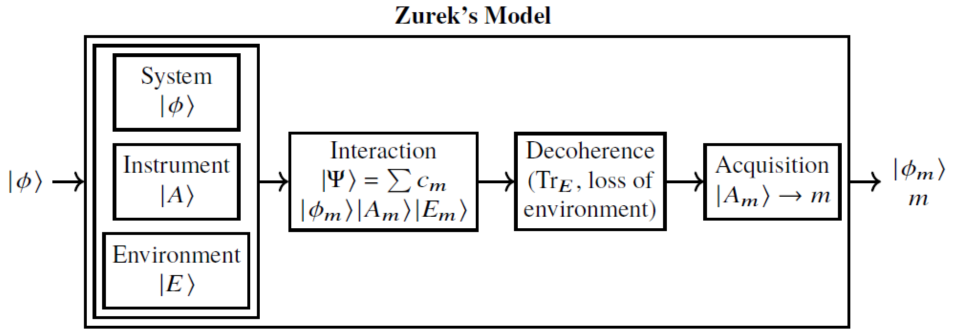
If the joint system with the instrument was isolated, we would expect the instrument giving the outcome to behave as a quantum mechanical pure state, while what we obtain in practice is a system behaving classically.

We can understand this behaviour by assuming that the measurement instrument is itself always coupled to an environment which is always lost. This model to describe measurement is sometimes called **Zurek's model** and the environmental process at the basis of this theory is called **decoherence**.

Decoherence is the loss of coherence, which is the superposition of the states that are to be measured.

Formally, a measurement implies not only a coupling between an original quantum system $|\psi\rangle$ and apparatus $|A\rangle$ but also a coupling with the environment $|E\rangle$ (considered a quantum-mechanical system).

From this perspective, it can also turn out that any quantum system is, in reality, an open system, because it does not exist in a completely isolated quantum system. After coupling the joint state becomes $|\psi\rangle|A\rangle|E\rangle$.



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Quantum Computing and Quantum Circuits

Single-Qubit Quantum Circuits

Qubit Gate	Circuit	Matrix
Hadamard	H	$H = \frac{1}{\sqrt{2}}\begin{bmatrix}1 & 1\\ 1 & -1\end{bmatrix}$ $\lvert a\rangle \rightarrow \frac{1}{\sqrt{2}}(\lvert 0\rangle + (-1)^a \lvert 1\rangle)$
Pauli X (Bit flip, NOT)	X	$X = \begin{bmatrix}0 & 1\\ 1 & 0\end{bmatrix}$ $\lvert a\rangle \rightarrow \lvert a \oplus 1\rangle$
Pauli Y (Bit&Phase flip)	Y	$Y = \begin{bmatrix}0 & -i\\ i & 0\end{bmatrix}$ $\lvert a\rangle \rightarrow i(-1)^a \lvert a \oplus 1\rangle$
Pauli Z (Phase flip)	Z	$Z = \begin{bmatrix}1 & 0\\ 0 & -1\end{bmatrix}$ $\lvert a\rangle \rightarrow (-1)^a \lvert a\rangle$
Phase gate (S or P gate)	S	$S = \begin{bmatrix}1 & 0\\ 0 & i\end{bmatrix}$ $\lvert a\rangle \rightarrow (-1)^a \lvert a\rangle$
Phase shift/rotation	$R(\alpha)$	$R(\alpha) = \begin{bmatrix}1 & 0\\ 0 & e^{i\alpha}\end{bmatrix}$ $\lvert a\rangle \rightarrow e^{ia\alpha} \lvert a\rangle$
Z Measurement		not a matrix $C \otimes \lvert a\rangle \rightarrow CC^{\dagger} \otimes \lvert a\rangle\langle a\rvert$
Serial gates	A_1 A_2	$A_2 A_1$

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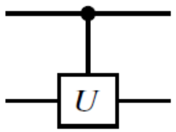
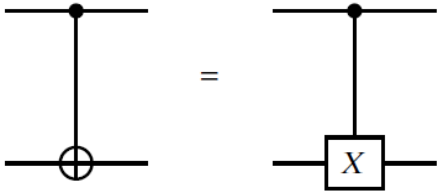
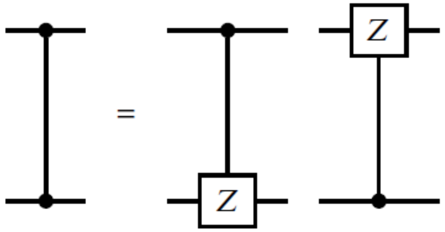
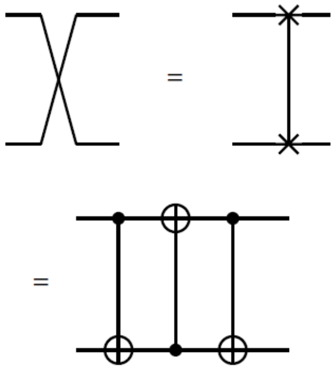
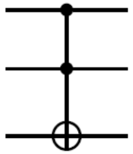
Two-Qubit Quantum Circuits

Multiqubit Gate	Circuit	Matrix
CU (Controlled U)		$\begin{bmatrix} 1 & 0 & 0 & 0 \\ 0 & 1 & 0 & 0 \\ 0 & 0 & \multicolumn{2}{c}{\multirow{2}{*}{U}} \\ 0 & 0 & & \end{bmatrix}$ $\lvert a, b\rangle \rightarrow \lvert a\rangle \otimes U^a \lvert b\rangle$
CNOT (Controlled X)		$\begin{bmatrix} 1 & 0 & 0 & 0 \\ 0 & 1 & 0 & 0 \\ 0 & 0 & 0 & 1 \\ 0 & 0 & 1 & 0 \end{bmatrix}$ $\lvert a, b\rangle \rightarrow \lvert a, a \oplus b\rangle$
CZ (Controlled Z)		$\begin{bmatrix} 1 & 0 & 0 & 0 \\ 0 & 1 & 0 & 0 \\ 0 & 0 & 1 & 0 \\ 0 & 0 & 0 & -1 \end{bmatrix}$ $\lvert a, b\rangle \rightarrow (-1)^{a \cdot b} \lvert a, b\rangle$
SWAP		$\begin{bmatrix} 1 & 0 & 0 & 0 \\ 0 & 0 & 1 & 0 \\ 0 & 1 & 0 & 0 \\ 0 & 0 & 0 & 1 \end{bmatrix}$ $\lvert a, b\rangle \rightarrow \lvert b, a\rangle$
Toffoli		$\begin{bmatrix} 1 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 1 & 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 1 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 1 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 1 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 1 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 & 0 & 1 \\ 0 & 0 & 0 & 0 & 0 & 0 & 1 & 0 \end{bmatrix}$ $\lvert a, b, c\rangle \rightarrow \lvert a, b, ab \oplus c\rangle$

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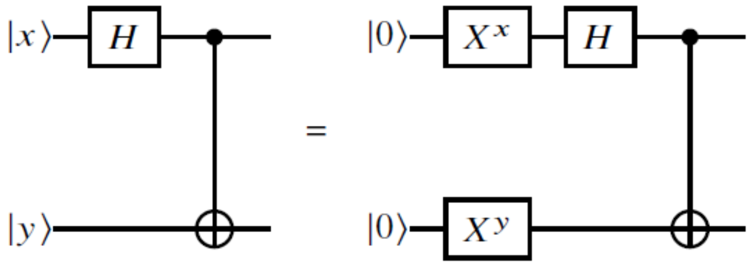
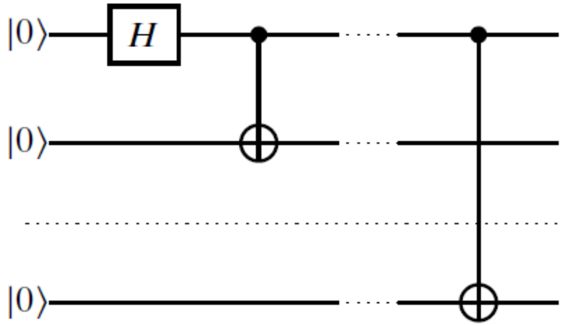
Quantum Circuits and Entanglement

Quantum circuits to represent entanglement.

State	Circuit
Bell states $\lvert\phi_{xy}\rangle = \frac{1}{\sqrt{2}}\sum_i(-1)^x \lvert i, y \oplus i\rangle$	
GHZ states $\lvert n\text{-GHZ}\rangle = \frac{1}{\sqrt{2}}(\lvert 0\rangle^{\otimes n} + \lvert 1\rangle^{\otimes n})$	

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Final Remarks

With quantum circuits, compared to classical circuits, some operations are prohibited.

Since the columns of the circuit represent time steps, loops in such circuits are not allowed.

Moreover, due to the **no-cloning theorem**, fan-in and fan-out of quantum systems are also not possible, since the information of one qubit cannot be copied into two qubits, and erasing a qubit is a noisy operation.

Preliminary Quantum Communication Protocols

Superdense Coding

Superdense coding involves two-communicating nodes, normally called Alice and Bob. Especially, Alice wants to send classical information to Bob.

Alice has two classical bits to be sent to Bob.

Alice and Bob share an EPR pair $|\Psi\rangle = \frac{|00\rangle+|11\rangle}{\sqrt{2}}$

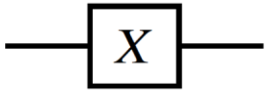
By sending her qubit to Bob, she can send two classical bits.

If she wants to send the bits 00, 01, 10, 11 she has to apply **I**, **Z**, **X** *i***Y** circuits respectively.

$$00: |\Psi\rangle = \frac{|00\rangle + |11\rangle}{\sqrt{2}} \qquad 01: |\Psi\rangle = \frac{|00\rangle - |11\rangle}{\sqrt{2}} \qquad 10: |\Psi\rangle = \frac{|10\rangle + |01\rangle}{\sqrt{2}} \qquad 11: |\Psi\rangle = \frac{|01\rangle - |10\rangle}{\sqrt{2}}$$

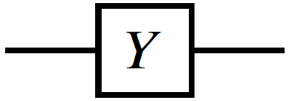
$$Z = \begin{bmatrix} 1 & 0 \\ 0 & -1 \end{bmatrix}$$

$$|a\rangle \rightarrow (-1)^a \, |a\rangle$$



$$X = \begin{bmatrix} 0 & 1 \\ 1 & 0 \end{bmatrix}$$

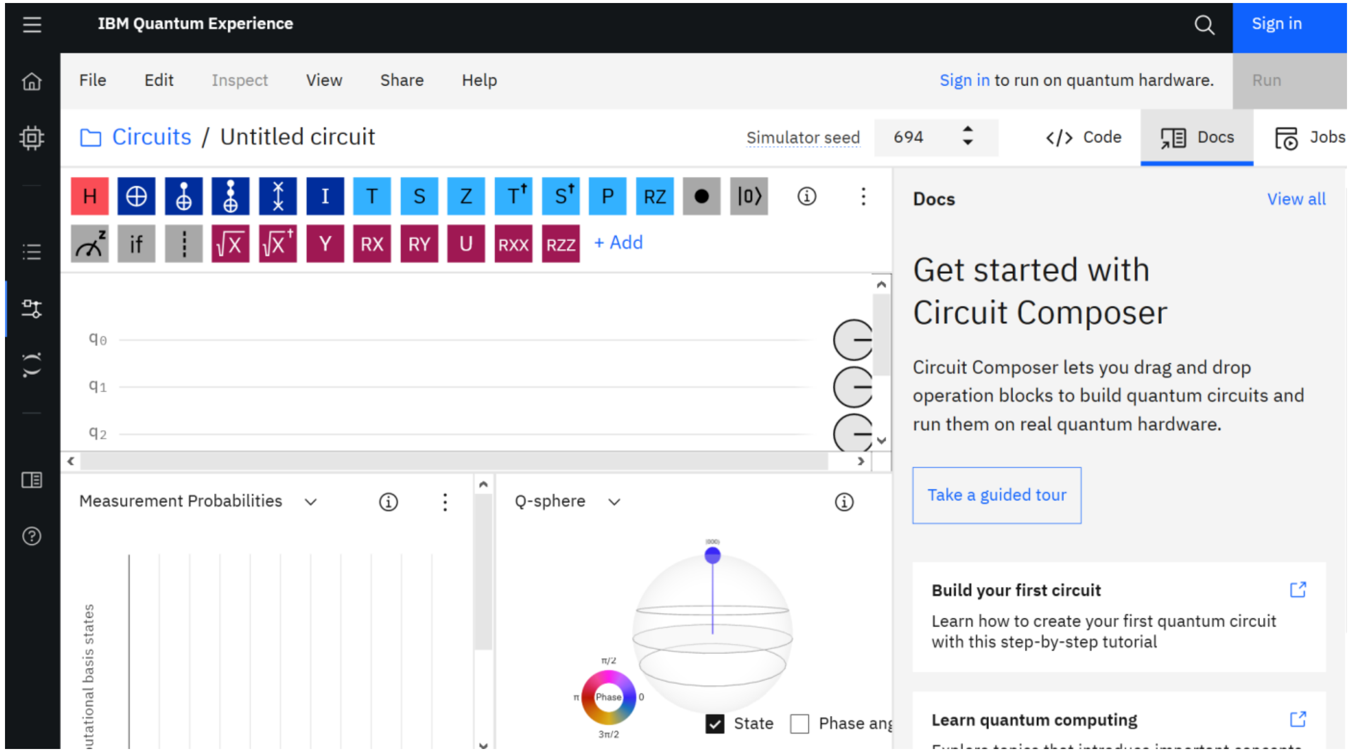
$$|a\rangle \rightarrow |a \oplus 1\rangle$$



$$Y = \begin{bmatrix} 0 & -i \\ i & 0 \end{bmatrix}$$

$$|a\rangle \rightarrow i(-1)^a \, |a \oplus 1\rangle$$

IBM Quantum Emulator and Simulator

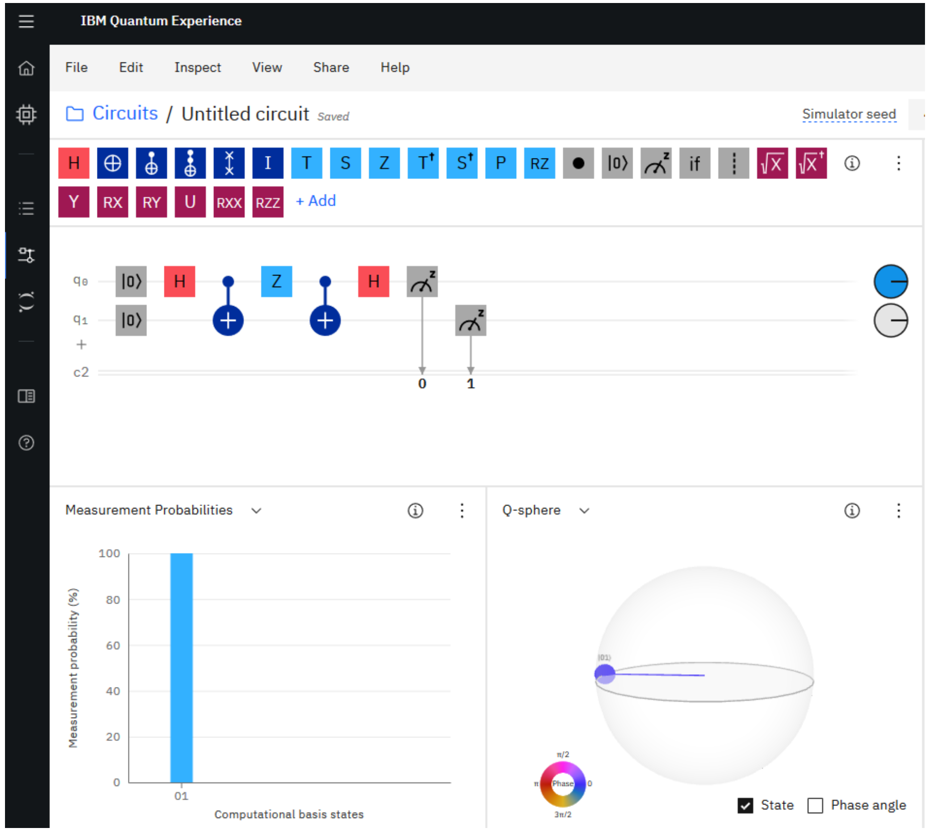


https://quantum-computing.ibm.com/

https://qiskit.org/textbook/preface.html

Superdense Coding

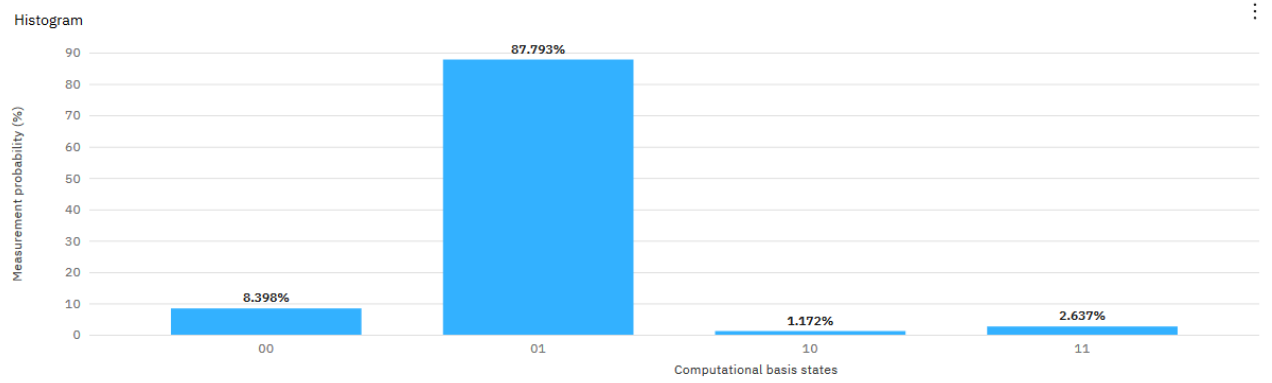
Theory

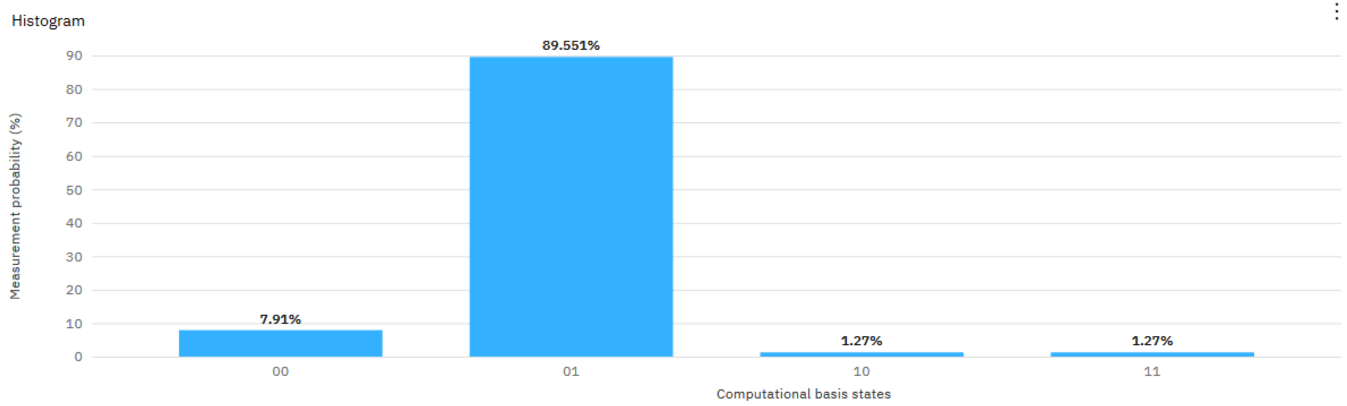


https://quantum-computing.ibm.com/

Practice

Result



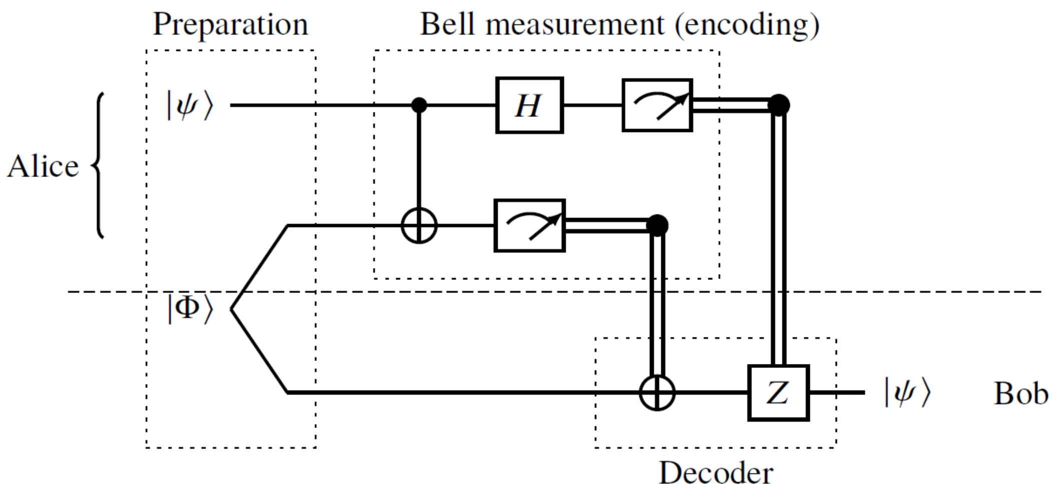


Teleportation

Quantum teleportation aims at sending of a quantum state from one place to another.

There are tow possible ways to send a quantum state:

- **Flying qubit** – Transporting the quantum object bodily from one place to another.
- **Teleportation** – Transportation of a quantum state by transmitting an ordinary classical information. However, it is not possible to transmit a quantum state using only classical signalling. The reason for this should be clear, because classical signals, by their very nature, can be copied. If they could be used to transmit a quantum state, then quantum states could also be copied, and we have seen above that this should not be possible.



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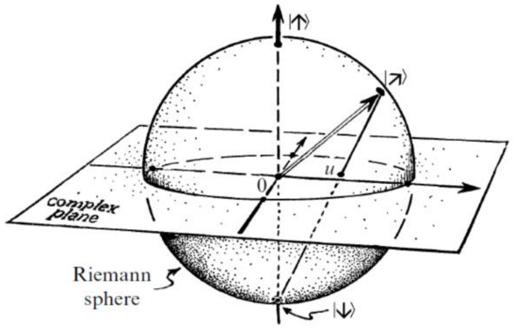
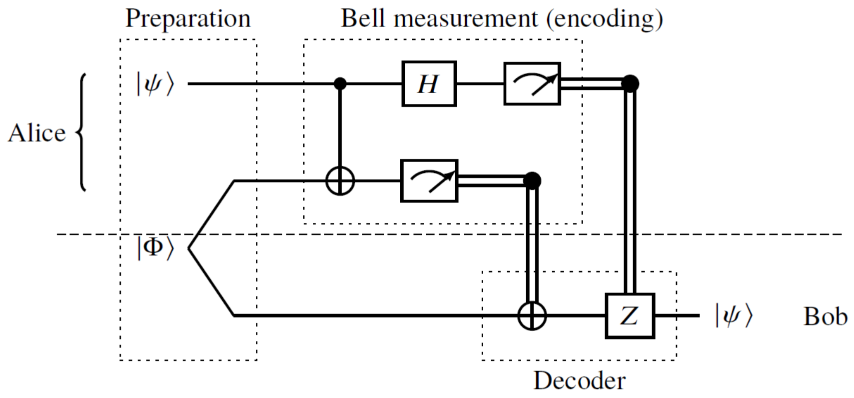
Teleportation

By sending merely 2 bits of classical information (one of the couples 00, 01, 10, 11 respectively), a quantum state (qubit) can be used to convey the ‘information’ of a point on the entire Riemann sphere.

In ordinary classical terms, this would have needed the information contained in the unrestricted choice of a point in a continuum: strictly $\aleph_0$ bits.





R. Penrose. The Road to Reality: A Complete Guide to the Laws of the Universe. Vintage Series. Vintage Books, 2007.

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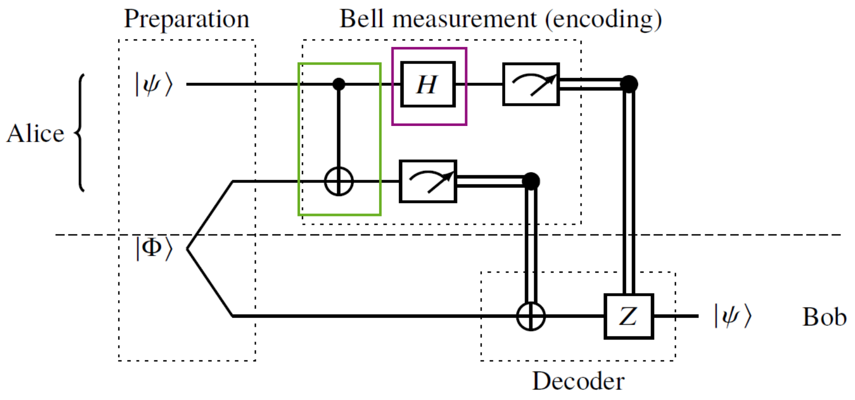
Teleportation



Alice and Bob share an EPR pair $|\Phi\rangle = \frac{|00\rangle + |11\rangle}{\sqrt{2}}$

Step 1: $|\psi\rangle \otimes |\Phi\rangle = \frac{c_1|0\rangle\otimes(|00\rangle + |11\rangle) + c_2|1\rangle\otimes(|00\rangle + |11\rangle)}{\sqrt{2}} = \frac{c_1|000\rangle + c_1|011\rangle + c_2|100\rangle + c_2|111\rangle}{\sqrt{2}}$

Step 2 [CNOT]: $|\psi'\rangle = \frac{c_1|000\rangle + c_1|011\rangle + c_2|110\rangle + c_2|101\rangle}{\sqrt{2}}$

Step 3 [H gate] $|\psi''\rangle = \frac{c_1|000\rangle + c_1|100\rangle + c_1|011\rangle + c_1|111\rangle + c_2|010\rangle - c_2|110\rangle + c_2|001\rangle - c_2|101\rangle}{2}$

$$|\psi''\rangle = \frac{|00\rangle\otimes(c_1|0\rangle + c_2|1\rangle) + |01\rangle\otimes(c_1|1\rangle + c_2|0\rangle) + |10\rangle\otimes(c_1|0\rangle - c_2|1\rangle) + |11\rangle\otimes(c_1|1\rangle - c_2|0\rangle)}{2}$$

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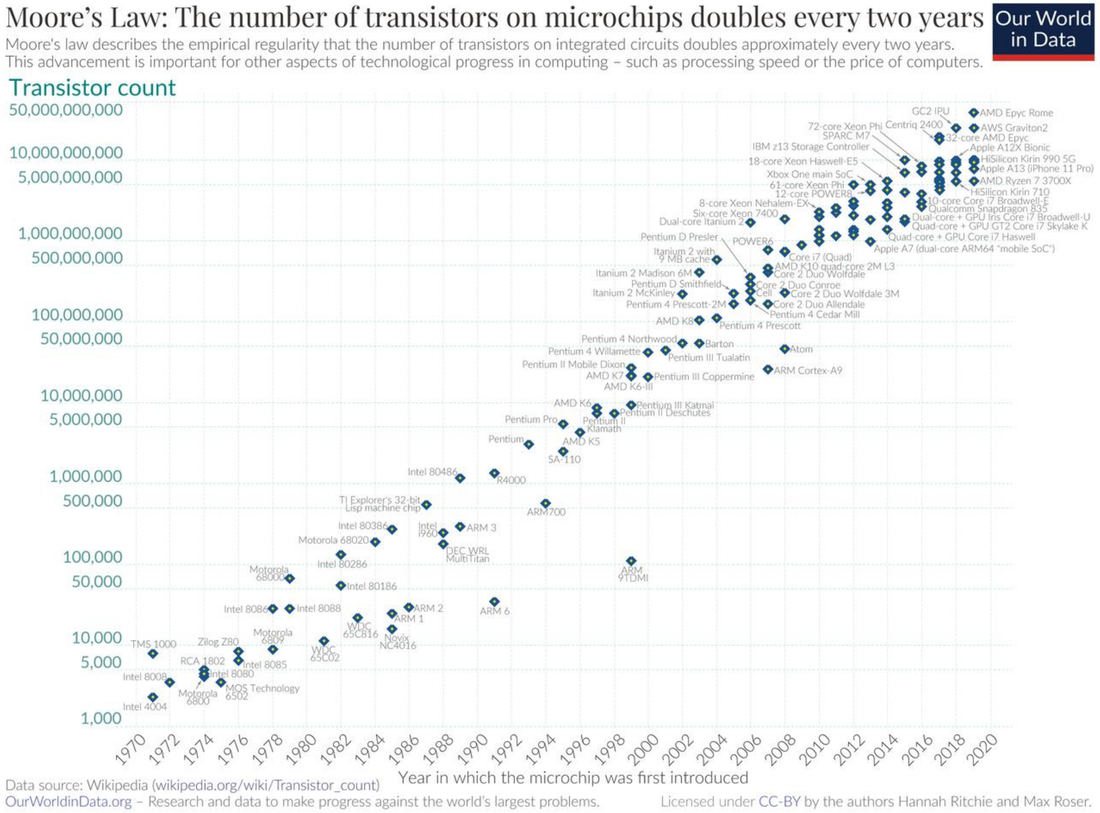
Why Quantum Communications and Why Now

The Moore’s Law

2019 – Samsung Electronics announced they had been offering their **5 nm** process (for example, used in current 2020-2021 processors of smartphones).



Semiconductor device fabrication

MOSFET scaling (process nodes)

- 10 µm – 1971
- 6 µm – 1974
- 3 µm – 1977
- 1.5 µm – 1981
- 1 µm – 1984
- 800 nm – 1987
- 600 nm – 1990
- 350 nm – 1993
- 250 nm – 1996
- 180 nm – 1999
- 130 nm – 2001
- 90 nm – 2003
- 65 nm – 2005
- 45 nm – 2007
- 32 nm – 2009
- 22 nm – 2012
- 14 nm – 2014
- 10 nm – 2016
- 7 nm – 2018
- 5 nm – 2020

Future

- 3 nm – *~2022*
- 2 nm – *>2023*

https://en.wikipedia.org/wiki/Moore's_law

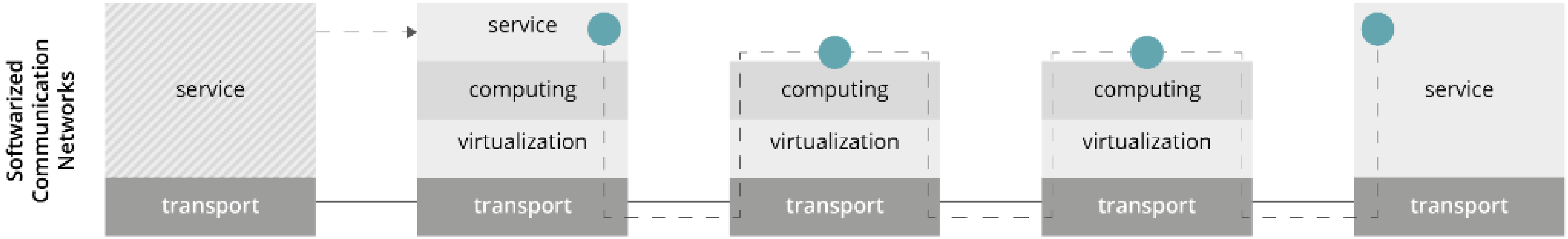
Wireless Communication Networks Towards 5G

Communication paradigms:

- **Circuit Switching**
  - Global System for Mobile Communications (**GSM**) is a digital, circuit-switched network for full duplex voice telephony.
- **Packet Switching**
  - General Packet Radio Service (**GPRS**), Enhanced Data rates for GSM Evolution (**EDGE**), Universal Mobile Telecommunications System (**UMTS**), and Long-Term Evolution (**LTE**).

R. Bassoli, H. Boche, C. Deppe, R. Ferrara, F. H. P. Fitzek, G. Janssen, S. Saeedinaeen, “Quantum Communication Networks”, 1st Ed., Springer, 2021, ISBN: 978-3-030-62938-0.

Wireless Communication Networks Towards 5G



5G and beyond vision – **compute-and-forward**:

- **Network virtualisation**
  - Software-defined Networking (**SDN**) and Network Function Virtualization (**NFV**)
- **Intelligent networks**
  - Supervised/unsupervised machine learning and reinforcement learning (thus autonomic networks).

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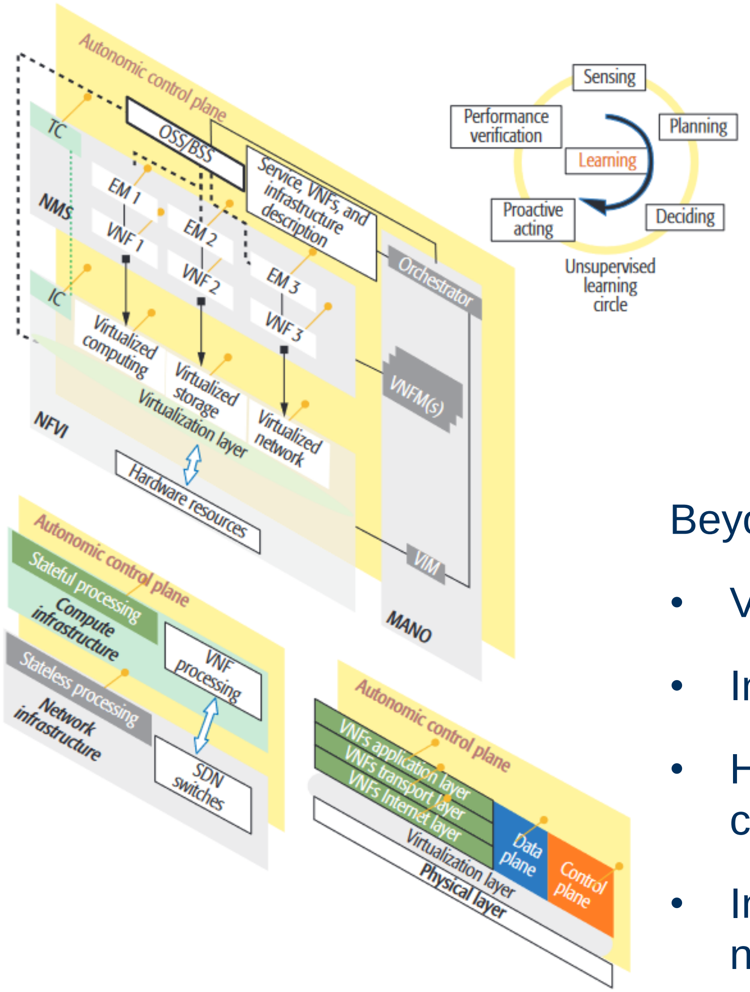
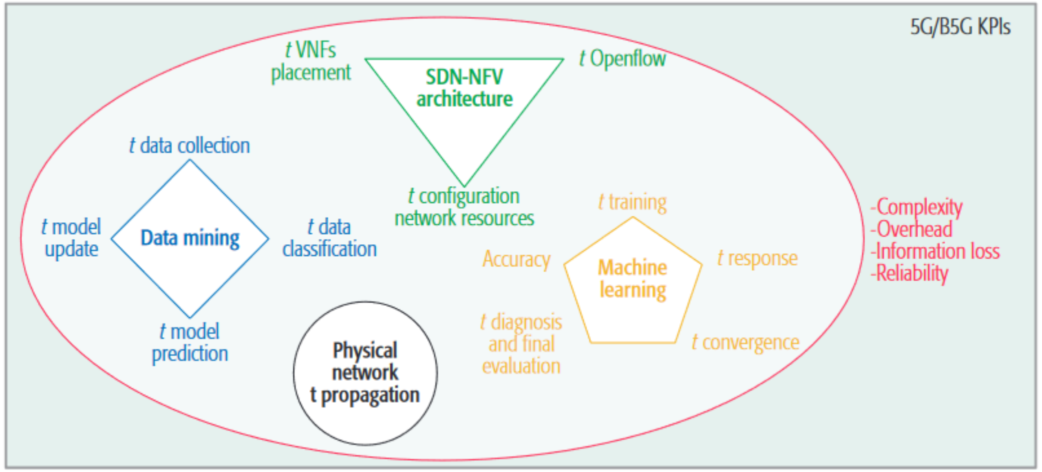
Communication Networks Beyond 5G





Beyond 5G vision – **Main drawbacks**:

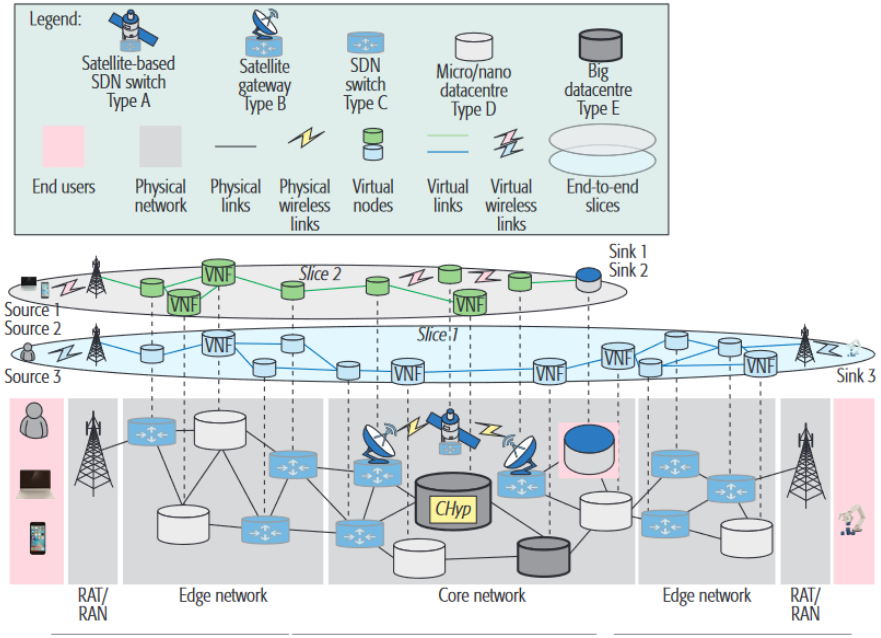
- Very high demand for storage and computing capacity.
- Increased energy consumption.
- Huge number of resources for secure data mining/processing and distributed computing for decision-making.
- Intelligent analysis of Big Data will continuously need network performance and network infrastructure awareness for prediction of future network states.

F. Granelli and R. Bassoli, "Autonomic Mobile Virtual Network Operators for Future Generation Networks," in IEEE Network, vol. 32, no. 5, pp. 76-84, September/October 2018, doi: 10.1109/MNET.2018.1700455.

Communication Networks Beyond 5G

Beyond 5G vision – **Main drawbacks**:

- **Software and Virtualisation**
  - Software abstraction introduces additional packet processing delays.
  - Packet I/O and processing operations of the virtual environments (virtual machines, containers, etc.) are slow considering the key-performance indicators (KPIs) of 5G.
  - More and more software is getting into network layers so that latency due to network virtualisation is expected to constantly increase.
  - End-to-end Latency = propagation latency + transmission delay + queuing delay + processing delay.



  - Reliability and resilience software functions are more prone to failures than hardware.
  - The placement of computing at a given data centre (e.g. big, micro, femto, etc.) has impact on resilience, capacity and latency.
  - The distributed nature of the in-network computing could also be extended to the user end device, adding more points of delay and failure.

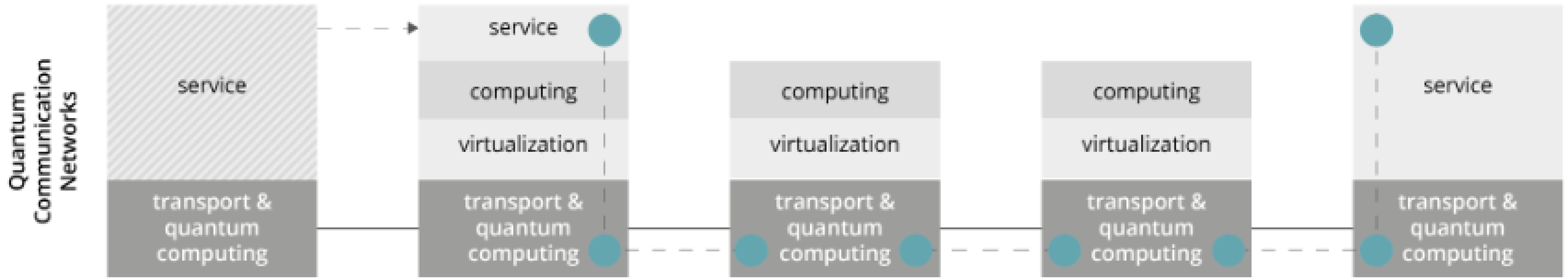
F. Granelli and R. Bassoli, "Autonomic Mobile Virtual Network Operators for Future Generation Networks," in IEEE Network, vol. 32, no. 5, pp. 76-84, September/October 2018, doi: 10.1109/MNET.2018.1700455.

Communication Networks Beyond 5G

Beyond 5G vision – **Main drawbacks**:

- **New security threats**
  - Threat due to scalability. Controllers and hypervisors can easily become bottlenecks because of the amount of control traffic they have to manage. Control plane saturation opens the door to various Denial-of-Service (DoS) and Distributed Denial-of-Service (DDoS) attacks.
  - Authenticating applications becomes fundamental. It is necessary to establish a trust relationship between the control plane and applications.
- **Energy consumption**.
  - The energy required by computing can be divided into two main categories: energy used by network/computing equipment (e.g., servers, networks, storage, etc.) and energy used by infrastructure facilities (such as cooling, air conditioning, etc.).
- **Communication complexity**.
  - Communication complexity is the amount of information (in terms of bits) that spatially separate computing devices need to exchange in order to successfully perform a computational task.
  - Mobile Edge Computing (MEC) and distributed computing for network functions will rely on distributed devices solving network-related computing problems. However, some of these distributed computing problems were demonstrated not to be solvable via distributed computing based on classical networks.

Towards Quantum Communication Networks

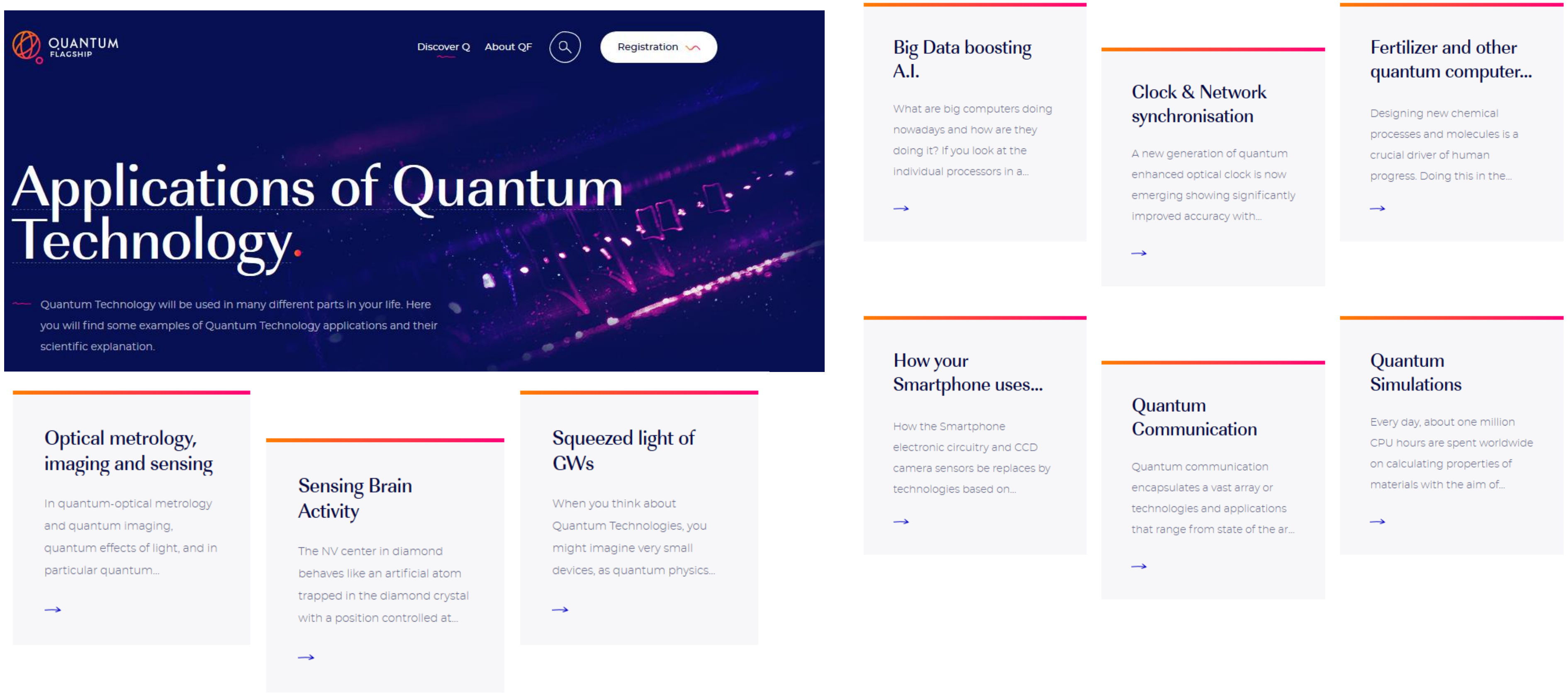


Hybrid classical-quantum network infrastructure.

- Nodes of the network can become distributed parts of the same physical system.
- This is possible because of Entanglement (Verschränkung).

R. Bassoli, H. Boche, C. Deppe, R. Ferrara, F. H. P. Fitzek, G. Janssen, S. Saeedinaeen, "Quantum Communication Networks", 1st Ed., Springer, 2021, ISBN: 978-3-030-62938-0.

The Quantum Flagship in Europe



https://qt.eu/discover-quantum/applications-of-qt/